MA 573 — PROJECT 1

Due: Wednesday, August 29

The goal of this project is to acquaint you with Matlab and LaTeX. In class we discussed the linear spring model

$$m\frac{d^2x}{dt^2}(t) + c\frac{dx}{dt}(t) + kx(t) = f(t)$$
$$x(0) = x_0 \ , \ \dot{x}(0) = v_0 \tag{1}$$

where m, c and k respectively denote the mass, damping and stiffness coefficients. We know that the analytic solution to (1) with $f(t) = 0$ is

$$x(t) = e^{-ct/2m}[A\cos(\nu t) + B\sin(\nu t)] \tag{2}$$

where

$$\nu = \frac{\sqrt{4km - c^2}}{2m} \ , \ A = x_0 \ , \ B = \left(x_1 + \frac{c}{2m}x_0\right)/\nu.$$

To numerically approximate the solution to (1), we discussed the implicit Euler algorithm

$$\vec{x}_{j+1} = [I - \Delta tA]^{-1}\vec{x}_j + [I - \Delta tA]^{-1}\Delta t\vec{F}(t_{j+1}) \tag{3}$$

where

$$A = \begin{bmatrix} 0 & 1 \\ -k/m & -c/m \end{bmatrix} \quad , \quad \vec{F}(t) = \begin{bmatrix} 0 \\ f(t)/m \end{bmatrix}$$

and $\vec{x} = [x\,,\,\dot{x}]^T$. The true and approximate solutions obtained with $m = 4, c = 2, k = 16$ and $x_0 = 2, x_1 = 30$ are plotted in Figure 1.

Assignment: Implement the trapezoidal method for the model (1) and write up your results in LaTeX. Compare the convergence rate of the trapezoidal method with that of the implicit Euler algorithm and include a plot of your results. You can work together on this but submit individual reports. If you are working on the eos/unity system, you need to add `tetex` to use LaTeX.

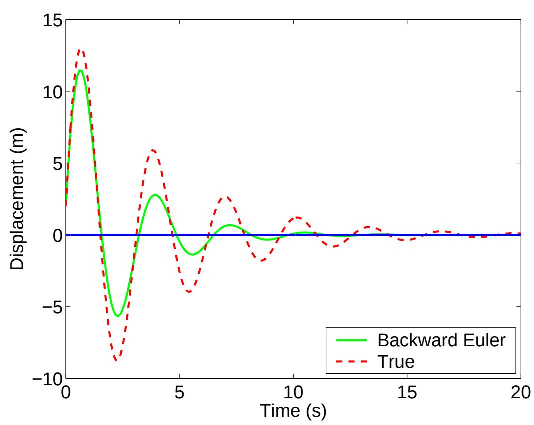

Figure 1: True and approximate solutions to the spring model (1).