Due Thursday, November 10

(1) Consider the system

\[
\dot{x} = Ax + Bu \\
y = Cx + Du
\]

where

\[
A = \begin{bmatrix}
3 & 6 & 4 \\
9 & 6 & 10 \\
-7 & -7 & -9
\end{bmatrix} \quad B = \begin{bmatrix}
-2/3 & 1/3 \\
1/3 & -2/3 \\
1/3 & 1/3
\end{bmatrix} \quad C = \begin{bmatrix}
1 & 2 & 3 \\
3 & 3 & 6
\end{bmatrix} \quad D = 0.
\]

(a) Show that the system is not controllable and use a QR decomposition to find a Kalman controllable canonical form.

(b) Show that the system is not observable and use a QR decomposition to find a Kalman observable canonical form.

(2) Consider the system

\[
\dot{x} = Ax + Bu \\
y = Cx.
\]

In the next chapter, we will study feedback systems in which the control is specified by

\[
u(t) = -Fx(t)
\]

where \(F\) is chosen to attain or enhance stability margins or achieve some other goal. Note that the resulting system has the form

\[
\dot{x} = (A - FB)x \\
y = Cx.
\]

One way to choose \(F\) is through a technique termed pole or eigenvalue placement which is facilitated by the companion controller form (further details can be found on pages 330-335 of our text). To illustrate, consider a system

\[
\dot{z} = \begin{bmatrix}
0 & 1 & 0 & \cdots & 0 \\
0 & 0 & 1 & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & \cdots & 1
\end{bmatrix} \begin{bmatrix}
\alpha_0 \\
-\alpha_1 \\
-\alpha_2 \\
\vdots \\
-\alpha_{n-1}
\end{bmatrix} + \begin{bmatrix}
0 \\
0 \\
\vdots \\
1
\end{bmatrix} u
\]

where the \(\alpha_i\)'s are the coefficients of the characteristic polynomial

\[
\alpha(s) = |sI - A| = s^n + \alpha_{n-1} s^{n-1} + \cdots + \alpha_1 s + \alpha_0.
\]

We now consider a feedback control of the form

\[
u(t) = -f_0 z_1 - f_1 z_2 - \cdots - f_{n-1} z_n = -F_c z.
\]
The closed loop system will now have the form

\[
\dot{x} = \begin{bmatrix}
0 & 1 & 0 & \cdots & 0 \\
0 & 0 & 1 & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & \cdots & 1 \\
-(\alpha_0 + f_0) & -(\alpha_1 + f_1) & -(\alpha_2 + f_2) & \cdots & -(\alpha_{n-1} - f_{n-1})
\end{bmatrix} x.
\]

Note that the signs of the \( f_i \) are opposite those in the book. This is to accommodate the more common convention of considering \( A - BF \), and yields the same final result.

Now, if the desired closed loop eigenvalues are specified by \( s^d_1, s^d_2, \ldots, s^d_n \), then the desired characteristic equation will be

\[
\Delta^d(s) = (s - s^d_1)(s - s^d_2) \cdots (s - s^d_n) = s^n + \alpha^d_{n-1}s^{n-1} + \cdots + \alpha^d_1s + \alpha^d_0
\]

from which it follows that

\[
\alpha_0 + f_0 = \alpha^d_0 \Rightarrow f_0 = \alpha^d_0 - \alpha_0 \\
\alpha_1 + f_1 = \alpha^d_1 \Rightarrow f_1 = \alpha^d_1 - \alpha_1 \\
\vdots
\]

\[
\alpha_{n-1} + f_{n-1} = \alpha^d_{n-1} \Rightarrow f_{n-1} = \alpha^d_{n-1} - \alpha_{n-1}
\]

Given specified eigenvalue locations, this provides an algorithm for computing the feedback gain \( F \). Note that if the system is not in companion form, previously discussed similarity transforms can be applied first to obtain this form.

(a) Consider the system

\[
\dot{z} = \begin{bmatrix}
0 & 1 & 0 \\
0 & 0 & 1 \\
-2 & -5 & -10
\end{bmatrix} z + \begin{bmatrix}
0 \\
0 \\
1
\end{bmatrix} u.
\]

Determine the eigenvalues for the system and discuss its stability. Now find a feedback \( F_c \) so that the closed loop system has eigenvalues located at \( s^d_{1,2} = -1 \pm i, s^d_3 = -5 \). Be sure to check the final system matrix to ensure that these values are obtained.

(b) Now consider the system

\[
\dot{x} = \begin{bmatrix}
1 & 2 & 0 \\
1 & -3 & 4 \\
-1 & 1 & -9
\end{bmatrix} x + \begin{bmatrix}
1 \\
2 \\
-1
\end{bmatrix} u.
\]

Again, check the stability of the system. Determine a transformation \( P \) which transforms the system to controller companion form. Finally, compute a feedback \( F = F_cP \) so that the closed loop system has eigenvalues at \(-1, -2, -3\). Note that the Matlab command \texttt{poly.m} can be used to find the characteristic polynomial of the matrix \( A \).