MA 573 — Project 2 Hints

The solution to the differential equation

$$m\ddot{y}(t) + c\dot{y}(t) + ky(t) = F\cos(\omega t) \tag{1}$$

is

$$y(t) = c_1 e^{r_1 t} + c_2 e^{r_2 t} + \frac{F}{\sqrt{(k - m\omega)^2 + (c\omega)^2}} \cos(\omega t - \delta) \tag{2}$$

where r_1 and r_2 are solutions to the characteristic equation and δ is the phase shift. For long time intervals, the transient solution $y_h(t) = e^{r_1 t} + c_2 e^{r_2 t}$ goes to 0 which leaves the steady solution

$$y(t) = \mathcal{F}(\omega)\cos(\omega t - \delta) \tag{3}$$

where

$$\mathcal{F}(\omega) = \frac{F}{\sqrt{(k - m\omega)^2 + (c\omega)^2}}.$$

You can assume that the beam driven by the patches has reached steady state at the time we stop applying voltage to the patches and begin data collection.

At steady state, the solution satisfies

$$\dot{y}(t) = -\omega\mathcal{F}(\omega)\sin(\omega t - \delta)$$

and

$$\begin{aligned} \ddot{y}(t) &= -\omega^2\mathcal{F}(\omega)\cos(\omega t - \delta) \\ &= -\omega^2 y(t). \end{aligned} \tag{4}$$

Hence the velocity and acceleration are 90^o and 180^o out-of-phase with the displacement.

For your project, you should consider the initial value problem starting at a point where the velocity is zero. You can determine the initial displacement using the relation (4). The patch input has been turned off so the solution is

$$y(t) = e^{-ct/2m}[A\cos(\nu t) + B\sin(\nu t)] \tag{5}$$

where

$$\nu = \frac{\sqrt{4km - c^2}}{2m}\ ,\ A = y_0\ ,\ B = \left(v_0 + \frac{c}{2m}y_0\right)/\nu$$

where y_0 and v_0 are the initial displacement and velocity (see Project 1). The modeled acceleration $\ddot{y}_{mod}$ used in your least squares functional can be obtained by differentiating (5).