Math 797V Project 3

Due: March 20, 2008

Problem 1.

In Project 2, you estimated the parameters $q = (\rho, \gamma, YI, CI, k_p)$ for a cantilever beam excited by a voltage spike to a surface mounted patch. In this problem, we are going to use our statistical analysis to determine error bounds for the parameters. You can use the parameters that you determined for Project 2 as your estimate $\hat{q}$.

(a) Plot the residuals resulting from the model with these parameters. Does it appear that your errors ε_j are iid and have zero mean? Remember that the residuals and errors differ but the former can be used to motivate the validity of hypotheses.

(b) Determine an estimate $\hat{s}^2$ for the variance σ_0^2 of the errors.

(c) Use finite differences to construct a coviance estimate $\widehat{\text{cov}(Q)}$ and use this to determine standard deviations for each of the parameters.

(d) Determine 95% confidence intervals for each parameter.

(e) Discuss the strengths and limitations of your model.

Problem 2.

In this problem, we will consider the modeling of heat generated during the hardening of cement. As detailed in [1], The variables are

y: heat evolved in calories/gram of cement
x_1: percentage of tricalcium aluminate
x_2: percentage of tricalcium silicate
x_3: percentage of tetracalcium aluminoferite
x_4: percentage of dicalcium phosphate.

Observations are summarized in Table 1.

Obs. No.	x_1	x_2	x_3	x_4	y
1	7	26	6	60	78.5
2	1	29	15	52	74.3
3	11	56	8	20	104.3
4	11	31	8	47	87.6
5	7	52	6	33	95.9
6	11	55	9	22	109.2
7	3	71	17	6	102.7
8	1	31	22	44	72.5
9	2	54	18	22	93.1
10	21	47	4	26	115.9
11	1	40	23	34	83.8
12	11	66	9	12	113.3
13	10	68	8	12	109.4

Table 1: Hald's cement data.

(a) Consider first the linear model

$$Y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \varepsilon. \tag{1}$$

Estimate the parameters, plot the residual, and determine confidence intervals of one standard deviation as well as 95% confidence intervals. Does it appear that the variables are significant at the 5% level?

(b) Perform the same analysis using linear models that incorporate only x_1 as well as x_1 and x_2. Which model do you recommend as best?

References

[1] G. Hald, *Statistical Theory with Engineering Applications*, John Wiley and Sons, New York, 1952.