

Math 797V Project 2

Due: February 28, 2008

Problem 1.

In this problem, you will use Euler-Bernoulli beam equations to model the transverse vibrations of a beam excited by a voltage spike to a PZT patch. Data will consist of displacement measurements obtained from a proximity sensor.

You will estimate the parameters $q = (\rho, \gamma, YI, CI, k_p)$ by minimizing the functional

$$J^N(q) = \sum_{k=1}^{N_d} [w^N(t_k, \bar{x}; q) - w_k]^2 \quad (1)$$

where N_d denotes the number of time samples, \bar{x} is the location of the proximity sensor, and w_k is the measured displacement data at time t_k . The minimization is carried out subject to $w^N(t, x)$ satisfying the beam equation derived in class.

Experimental Procedure:

- (i) Carefully note details regarding the experimental setup. Do the end-clamps appear tight? Do you note any vibrations in the supporting structure? Does the beam appear uniform and homogeneous?
- (ii) Measure the dimensions of the beam as well as the location of the patch and proximity sensor. Collect displacement data.

Model Analysis and Parameter Estimation:

- (i) Write down weak formulations of the beam model that incorporate air damping, Kelvin-Voigt damping and an input force due to a voltage spike.
- (ii) Download the beam software from the website

<http://www.ncsu.edu/crsc/events/ugw07/schedule.php>

to approximate the beam displacement. One of the code options is to approximate the beam response to an input voltage. You will need to embed the code in an optimization routine to estimate the parameters through a least squares fit to the data.

This code discretizes the beam model using the basis functions

$$\phi_j(x) = \begin{cases} \hat{\phi}_0(x) - 2\hat{\phi}_{-1}(x) - 2\hat{\phi}_1(x) & , \quad j = 1 \\ \hat{\phi}_j(x) & , \quad j = 2, \dots, N + 1 \end{cases}$$

constructed from the cubic splines

$$\hat{\phi}_j(x) = \frac{1}{h^3} \begin{cases} (x - x_{j-2})^3, & x \in [x_{j-2}, x_{j-1}) \\ h^3 + 3h^2(x - x_{j-1}) + 3h(x - x_{j-1})^2 - 3(x - x_{j-1})^3, & x \in [x_{j-1}, x_j) \\ h^3 + 3h^2(x_{j+1} - x) + 3h(x_{j+1} - x)^2 - 3(x_{j+1} - x)^3, & x \in [x_j, x_{j+1}) \\ (x_{j+2} - x)^3, & x \in [x_{j+1}, x_{j+2}) \\ 0, & \text{otherwise.} \end{cases}$$

The beam displacements are approximated through the expansion

$$w^N(t, x) = \sum_{j=1}^{N+1} w_j(t) \phi_j(x)$$

in the space $V^N = \text{span}\{\phi_j\} \subset V = H_0^2(0, \ell)$.

- (iii) Estimate the parameters for the beam.
- (iv) Your writeup should contain a full discussion of your investigation as well as a summary of the estimated parameters and the model fit obtained with those parameters. A good way to illustrate the latter is by plotting the experimental data and model response together on the same axes. Be sure to state all conclusions regarding the accuracy and applicability of the model.

Problem 2. Consider the HIV model

$$\begin{aligned} \dot{T}_1 &= \lambda_1 - d_1 T_1 - (1 - \epsilon) k_1 V T_1 \\ \dot{T}_2 &= \lambda_2 - d_2 T_2 - (1 - f\epsilon) k_2 V T_2 \\ \dot{T}_1^* &= (1 - \epsilon) k_1 V T_1 - \delta T_1^* - m_1 E T_1^* \\ \dot{T}_2^* &= (1 - f\epsilon) k_2 V T_2 - \delta T_2^* - m_2 E T_2^* \\ \dot{V} &= N_T \delta (T_1^* + T_2^*) - cV - [(1 - \epsilon) \rho_1 k_1 T_1 + (1 - f\epsilon) \rho_2 k_2 T_2] V \\ \dot{E} &= \lambda_E + \frac{b_E (T_1^* + T_2^*)}{T_1^* + T_2^* + K_b} E - \frac{d_E (T_1^* + T_2^*)}{T_1^* + T_2^* + K_d} E - \delta_E E \end{aligned} \tag{2}$$

from Project 1. In this project, we are going to use the “Method of Nearby Problems” to establish that our ODE solver is providing the correct convergence rate (see Lecture 4).

We will be simulating for 150 days and fitting a spline every 10 days using the `spline` command. This can be accomplished for the variable `T1` using the commands

```
tspline = 0:10:150;
T1_spline = spline(t,T1,tspline);
pp = spline(tspline,T1_spline);
[breaks,coefs,l,k,d] = unmkpp(pp);
```

Here `coefs` contains the spline coefficients at each break point.

Construct the “nearby” or “pseudoproblem” and simulate the solutions using the ODE solver of your choice. Verify that you are obtaining the correct convergence rate for the pseudoproblem. Discuss the ramifications regarding the convergence rate for the original problem.