

Verification Techniques for PDE

Verification Strategies for PDE

Strategies:

- Method of manufactured solutions
- Compare multiple methods
- Compare multiple stepsizes
- Method of nearby problems
- Compare to analytic analytic solution or special cases

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Example: Siko-Streifer population model that groups individuals with common traits: $u(t,x)$ is size density at time t

$$\frac{\partial u}{\partial t} + \frac{\partial}{\partial x}(gu) = -\mu u, \quad x \in (x_0, x_1), t > 0$$

$$gu|_{x_0} = \int_{x_0}^{x_1} k(t, \xi)u(t, \xi)d\xi$$

$$gu|_{x_1} = 0$$

$$u(0, x) = f(x)$$

Methods:

- Characteristics (analytic)
- Finite differences
- Finite elements
- Construct solution
- Employ biological analysis

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Example: Heat model

$$\frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial x^2} + x(x-2) \sin(3\pi t)$$

$$u(t, 0) = u(t, 2) = 0$$

$$u(0, x) = \sin(\pi x/2)$$

One Strategy: Consider first

$$\frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial x^2}$$

$$u(t, 0) = u(t, 2) = 0$$

$$u(0, x) = \sin(\pi x/2)$$

that has the analytic solution

$$u(t, x) = \sin(\pi x/2) e^{-\alpha \pi^2 t/4}$$