Math 731 Project 4

Due Friday, April 24

(4) The dynamics of a DC motor are modeled by the system

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -x_2 + u$$

where the constraint $|u| \leq 1$ is placed on the control input. The goal is to drive states to the origin in *minimum time*.

(a) Determine the maximum number of times that the control can switch during a general interval $[0, t_f]$ and write the general form of the control law. Formulate the state and costate equations but do not solve.

(b) Eliminate the time variable by forming $\frac{dx_2}{dx_1} = \frac{\dot{x}_2}{\dot{x}_1}$. Compute and plot trajectories in the x_1x_2-plane for $u = 1$ and $u = -1$.

(c) Determine the equations of the switching curves which end at the origin. For initial values on these curves, formulate the control signal in feedback form.

(d) Sketch the minimum time path for the initial condition $(x_1, x_2) = (1, 1)$.

(2) Work through and write up Example 6.3-2, *Optimal Control by Solution of JHB Equation*, in Lewis and Syrmos. In particular, make sure that you are comfortable with the partial derivative relation for $\frac{\partial J^*}{\partial t}$. You do not need to provide any more detail than the book ... so remember this question when doing the next one :)

(3) Please make sure that you have done class evaluations (sorry, I cannot give any credit on this one :).