Project 3

1a) Shock should move to the left with speed determined by your parameter choices.

6) The solution along characteristic curves

\[ \frac{dx}{dt} = \frac{u}{\max} \left(1 - \frac{2p}{\rho_{\text{max}}} \right) \]

is constant. Thus

\[ \frac{dx}{dt} = \begin{cases} 
\frac{u}{\max}, & p > p_r \\
-\frac{u}{\max}, & p < p_r 
\end{cases} \]

2. )
Because the flow is steady, the continuity equation is

\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \Rightarrow \frac{\partial v}{\partial y} = 0 \text{ since } u = w = 0. \]

However, \( \frac{\partial u}{\partial x} = 0 \) so \( p = p(y, z) \). The \( y \)-component of the Navier-Stokes equations is

\[ \frac{\partial p}{\partial y} = \mu \left( \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right) = \frac{\partial p}{\partial y} = \mu \frac{\partial^2 v}{\partial y^2} \]

\[ \Rightarrow v(y) = \frac{1}{2 \mu} \frac{\partial p}{\partial y} y^2 + c_1 z + c_2 \]

\[ \Rightarrow v(y) = \frac{1}{2 \mu} \frac{\partial p}{\partial y} (z^2 - 2h) + \frac{V_0}{h} y. \]

Now,

\[ p = \mu \frac{\partial^2 v}{\partial y^2} y + h \]

Note: \( T_{xz} = T_{xy} = 0 \) since \( v = v(z) \) and \( u = w = 0 \). Moreover,

\[ T_{yz} = \mu \frac{\partial v}{\partial z} = \frac{1}{2} \frac{\partial p}{\partial y} (z^2 - h) + \frac{V_0}{h}. \]