MA 574 — Project 5

Due: Wednesday, April 30

Terfenol-D Transducer Model:

Magnetostrictive transducers similar to that depicted in Figure 1 are being considered for applications ranging from sonar transduction to high speed milling. In this project, you will model the displacement $u(t,\ell)$ of the Terfenol-D rod tip as a function of input currents $I(t)$ to the solenoid. The prestress mechanism adds an end stiffness of k_ℓ and damping c_ℓ while the end mass can be denoted by m_ℓ. You can assume that the the field H and magnetization M are related by the linear relation $M = \chi H$ where χ denotes the susceptibility. Moreover, you can assume that $H(t) = nI(t)$ where n denotes the number of turns per inch in the solenoid.

(1) Derive a strong form of the modeling PDE (distributed model) which specifies displacements $u(t,x)$ along the length of the rod. Include Kelvin-Voigt damping C to incorporate internal damping.

(2) Derive a lumped (ODE) model of the form

$$m\frac{d^2u_\ell}{dt^2}(t) + c\frac{du_\ell}{dt}(t) + ku_\ell(t) = \alpha I(t)$$

to specify the displacement $u_\ell = u(t,\ell)$ of the rod tip. Relate the parameters m, c, k to the original parameters $\rho, Y, C, A, m_\ell, k_\ell, c_\ell$. What assumptions are you making to motivate the accuracy of the ODE model?

(3) Take $C = c_\ell = I = 0$ and use Hamiltonian principle to derive a weak form of the PDE model that incorporates the effects of the boundary stiffness k_ℓ and mass m_ℓ. Now provide a general weak formulation which incorporates the effects of damping and input currents. Demonstrate the equivalence between the strong and weak solutions if they exhibit sufficient smoothness.

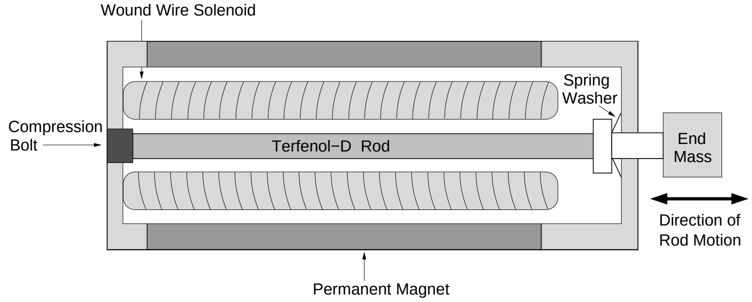

Figure 1: Cross section of a prototypical Terfenol-D transducer.