Math 574 Project 2

Due: February 27, 2008

Problem 1.

In this problem, you will use Euler-Bernoulli beam equations to model the transverse vibrations of a beam excited by a voltage spike to a PZT patch. Data will consist of displacement measurements obtained from a proximity sensor.

You will estimate the parameters $q = (\rho, \gamma, YI, CI, k_p)$ by minimizing the functional

$$J^N(q) = \sum_{k=1}^{N_d} \left[w^N(t_k, \bar{x}; q) - w_k \right]^2 \tag{1}$$

where N_d denotes the number of time samples, $\bar{x}$ is the location of the proximity sensor, and w_k is the measured displacement data at time t_k. The minimization is carried out subject to $w^N(t,x)$ satisfying the beam equation derived in class.

Experimental Procedure:

(i) Carefully note details regarding the experimental setup. Do the end-clamps appear tight? Do you note any vibrations in the supporting structure? Does the beam appear uniform and homogeneous?

(ii) Measure the dimensions of the beam as well as the location of the patch and proximity sensor. Collect displacement data.

Model Analysis and Parameter Estimation:

(i) Write down weak formulations of the beam model that incorporate air damping, Kelvin–Voigt damping and an input force due to a voltage spike.

(ii) Download the beam software from the website

`http://www.ncsu.edu/crsc/events/ugw07/schedule.php`

to approximate the beam displacement. One of the code options is to approximate the beam response to an input voltage. You will need to embed the code in an optimization routine to estimate the parameters through a least squares fit to the data.

This code discretizes the beam model using the basis functions

$$\phi_j(x) = \begin{cases} \widehat{\phi}_0(x) - 2\widehat{\phi}_{-1}(x) - 2\widehat{\phi}_1(x) & , \quad j = 1 \\ \widehat{\phi}_j(x) & , \quad j = 2, \ldots, N+1 \end{cases}$$

constructed from the cubic splines

$$\widehat{\phi}_j(x) = \frac{1}{h^3} \begin{cases} (x - x_{j-2})^3 \,, & x \in [x_{j-2}, x_{j-1}) \\ h^3 + 3h^2(x - x_{j-1}) + 3h(x - x_{j-1})^2 - 3(x - x_{j-1})^3 \,, & x \in [x_{j-1}, x_j) \\ h^3 + 3h^2(x_{j+1} - x) + 3h(x_{j+1} - x)^2 - 3(x_{j+1} - x)^3 \,, & x \in [x_j, x_{j+1}) \\ (x_{j+2} - x)^3 \,, & x \in [x_{j+1}, x_{j+2}) \\ 0 \,, & \text{otherwise.} \end{cases}$$

The beam displacements are approximated through the expansion

$$w^N(t,x) = \sum_{j=1}^{N+1} w_j(t)\phi_j(x)$$

in the space $V^N = \text{span}\{\phi_j\} \subset V = H_0^2(0,\ell)$.

(iii) Estimate the parameters for the beam.

(iv) Take the fft of the data and model response and compare them on one plot (you can use the code on the website `http://www4.ncsu.edu/~rsmith/MA573_f07.html` as a template). Are you accurately approximating the frequencies?

(v) Your writeup should contain a full discussion of your investigation as well as a summary of the estimated parameters and the model fit obtained with those parameters. A good way to illustrate the latter is by plotting the experimental data and model response together on the same axes. Be sure to state all conclusions regarding the accuracy and applicability of the model.

Problem 2. Measure the length, width and thickness of the second vertical beam. Observe and record the first 3 natural frequencies along with the modal shapes.

Now use separation of variables, with no damping, to show that the spatial component yields the relation

$$\cos(\xi) = \frac{-1}{\cosh(\xi)}$$

where $\xi = (\beta\rho/YI)^{1/4}$ and β is a constant to be determined. Plot the function and use the MATLAB command `fzero` to compute the first four values of ξ. Using the experimental frequencies, estimate the parameters ρ and YI. Do your values appear reasonable?