MA 114-005
Quiz 4(B)

Name: 
Row #: 

Directions: Show your work.

(a) Suppose that the population migration between two geographical regions—say, the North and the South—is as follows. Each year, 75% of the population in the North migrates to the South, while only 25% of the population in the South moves to the North. Write down the transition matrix for this situation. (5 pts)

\[
\begin{bmatrix}
\frac{3}{4} & \frac{1}{4} \\
\frac{1}{4} & \frac{3}{4}
\end{bmatrix}
\]

(b) If this migration pattern continues, what percentage of population will stabilize in the South? (5 pts)

Let \( x = \) population probability living in North, \( y = \) population probability living in South.

\[ S = (x, y) \]

\( S \mathbf{T} = S \) with \( x + y = 1 \)

\[
\begin{bmatrix}
x \\
y
\end{bmatrix}
\begin{bmatrix}
\frac{3}{4} & \frac{1}{4} \\
\frac{1}{4} & \frac{3}{4}
\end{bmatrix}
\begin{bmatrix}
x \\
y
\end{bmatrix}
\]

\[
\frac{3}{4}x + \frac{1}{4}y = x \Rightarrow -\frac{3}{4}x + \frac{1}{4}y = 0 \Rightarrow \begin{bmatrix}
-\frac{3}{4} & 1 & 0 \\
\frac{1}{4} & -1 & 1
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
1
\end{bmatrix}
\]

Use Gauss-Jordan

\[
\begin{bmatrix}
1 & 0 & \frac{1}{4} \\
0 & 1 & \frac{3}{4}
\end{bmatrix}
\Rightarrow \begin{bmatrix}
\frac{3}{4} & \frac{1}{4} & 0 \\
\frac{1}{4} & -\frac{1}{4} & 1
\end{bmatrix}
\Rightarrow \begin{bmatrix}
x \\
y \\
1
\end{bmatrix}
\]

\[ x = \frac{1}{4}, \quad y = \frac{3}{4} = 0.75 \]

75% will stabilize in the South.