MA 114-005
Quiz 4(A)
Name:
Row #:

Directions: Show your work

(a) Suppose that the population migration between two geographical regions—say, the North and the South is as follows. Each year, 50% of the population in the North migrates to the South, while only 25% of the population in the South moves to the North. Write down the transition matrix for this situation. (5 pts)

\[
T = \begin{pmatrix}
\frac{1}{4} & \frac{3}{4} \\
\frac{1}{2} & \frac{1}{2}
\end{pmatrix}
\]

(b) If this migration pattern continues, what percentage of population will stabilize in the North? (5 pts)

Let \( x \) = fraction of population stabilizing in North
\( y \) = fraction of population stabilizing in South

\[ S = \begin{pmatrix} x \\ y \end{pmatrix} \]

\[ ST = S \quad \text{with} \quad x + y = 1 \]

\[
\begin{pmatrix} x \\ y \end{pmatrix} \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{4} & \frac{3}{4} \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}
\]

\[
\frac{1}{2}x + \frac{1}{4}y = x \quad \Rightarrow \quad -\frac{1}{2}x + \frac{1}{4}y = 0 \quad \Rightarrow \quad \frac{1}{2}x = \frac{1}{4}y \quad \Rightarrow \quad y = 2x
\]

\[
\frac{1}{2}x + \frac{3}{4}y = y \quad \Rightarrow \quad \frac{1}{2}x - \frac{1}{4}y = 0 \quad \Rightarrow \quad x + y = 1
\]

Almost 33.33% will stabilize in North.

Note: You could also solve the system by the Gaussian Jordan method.

\[
\begin{pmatrix}
\frac{1}{2} & \frac{1}{4} & 0 \\
\frac{1}{2} & \frac{1}{4} & 1
\end{pmatrix}
\]

Gaussian Jordan

\[
\begin{pmatrix}
1 & 0 & \frac{1}{3} \\
0 & 1 & \frac{2}{3}
\end{pmatrix}
\]