6.4 Definitions and concepts

An absorbing Markov Chain is

An absorbing state is a state in which it is impossible to leave the state, i.e., the probability of an absorbing state is always 1.

A Markov chain is absorbing if (i) it has at least one absorbing state (ii) from each non-absorbing state, system can go to an absorbing state. It cannot take more than one step through.

When we have an absorbing Markov chain, then we rewrite the transition matrix.

We list absorbing states first and non-absorbing states later.

Our transition matrix takes the following structure now.

\[
T = \begin{pmatrix}
I & 0 \\
R & Q
\end{pmatrix}
\]

This form is known as Canonical form. If there are m absorbing states then matrix I (Identity matrix) is mxm. Q is also a square matrix. It's size is (n-m)x(n-m). n is total number of states. R has size (n-m)xm.

We can find the following in an absorbing Markov chain.

(i) average (expected value) of

(ii) number of times (of steps) the process is in a non-absorbing state if it began in a non-absorbing state.

(ii) total step average (of steps) for a non-absorbing state until it is absorbed.

(expected value)

(3) probabilities that the process ends up in an absorbing state.
If it started in a non-absorbing state.

Expected values:

1) and 2) are given by the following method.

If \( Q \) is \((n-m) \times (n-m)\), Define \( N = (I - Q)^{-1} \). Here the size of \( I \) (identity matrix) is same as size of \( Q \).

You can find the inverse by elementary row operations (method you learned in Ch 1).

1. Entries of \( N \) are expected values beginning from non-absorbing ending at non-absorbing states.

2. If we sum along a row \( i \) then this is the average number of times the system will be in row \( i \) until it is absorbed.

Form \( NR \). (multiply both matrices \( N \) and \( R \)).

The entries of \( NR \) are probabilities given in (3).

\[
C_{i,j} (Q \# 23) \quad \begin{pmatrix}
0 & 0 & 
\frac{1}{2} & 
\frac{1}{2} \\
0 & 0 & 
\frac{1}{2} & 
\frac{1}{2} \\
0 & 0 & 
\frac{1}{3} & 
\frac{1}{3} \\
0 & 0 & 
\frac{1}{3} & 1
\end{pmatrix}
\]

Note that 3rd row 3rd column entry = 1, 5th row 5th column entry = 1

these are absorbing states while

Canonical form:

\[
\begin{pmatrix}
\frac{1}{3} & 3 & 1 & 2 & 4 \\
\frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\
2 & \frac{1}{2} & 0 & \frac{1}{3} & 0 \\
\frac{1}{2} & 0 & 0 & \frac{1}{3} & 1
\end{pmatrix}
\]

\( N = (I - Q)^{-1} = 1 \begin{pmatrix}
\frac{3}{4} & \frac{1}{4} & \frac{3}{4} \\
\frac{1}{4} & \frac{3}{4} & \frac{3}{4} \\
\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\
\frac{1}{2} & \frac{1}{2} & \frac{1}{2}
\end{pmatrix}
\]

For (4) find \( NR \) = 2

\[
\begin{pmatrix}
3 & 5 & 5 \\
\frac{3}{8} & \frac{7}{8} & \frac{3}{8} \\
\frac{5}{8} & \frac{3}{8} & \frac{3}{8}
\end{pmatrix}
\]