

(*) **Instructions:** Show all work for full credit. Do all work in the Blue Books. You will have 180 minutes to complete this exam.

(*) **Page One:** Work all three problems.

1. (20) Definitions and Concepts.

- Consider $f(x) = \frac{(x-4)(x-6)}{(x-5)}$. Does $f(x)$ have a **hole** or **vertical asymptote** at $x = 5$?
- While the slope of the secant line gives the **average rate of change** between two points on a function, the slope of the tangent line gives _____.
- Draw a graph which has a single **critical point**, but not a local minimum or maximum.
- Can a function be both concave up and decreasing on its domain? If so, draw the graph of such a function.
- Can a function be continuous on its domain, but not differentiable on its domain? If so, draw the graph of such a function.
- Determine a function which satisfies the relationship $\frac{dP}{dt} = -kP$.
- Determine a function which satisfies the relationship $P'(t) = kP(t)$.
- Write a function f which satisfies the relationship $\frac{df}{dt} = \frac{1}{x}$.
- Suppose $\int_0^{\infty} f(x)dx$ **diverges**. Briefly explain what that means.
- Suppose I differentiate $f(x, y, z)$ with respect to y . What does that imply about variables x and z ?

2. (40) Differentiate each of the following:

- $f(x) = 5x^3 + \sqrt{x}$
- $f(x) = (x^5 + 1)^{10}$
- $f(x) = x^3 e^x$
- $f(x) = \frac{x^2+1}{2x-6}$
- $f(x) = e^{x^2+1}$
- $f(x) = \ln(x^2 + x + 5)$
- $f(x) = 5^{3x+1}$
- $f(x) = \log_4(3x)$

3. (40) Compute each of the following:

- $\int 6x^3 + 6\sqrt{x}dx$
- $\int \frac{5}{x} dx$
- $\int e^x + 2e^x dx$
- $\int \frac{6x^2+5}{2x^3+5x+1} dx$
- $\int 6xe^{x^2+1} dx$
- $\int 15x^2(x^3 + 2)^9 dx$
- $\int_0^1 x^2 + x dx$
- $\int_1^2 \frac{x-1}{x^2-x} dx$

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Page Two: Answer Five of Six.

If you work all six, clearly indicate which is to not be graded.

4. (20) Consider $f(x) = x^3 - 18x^2 + 105x - 57$.
 - a. Find all the relative extrema. Classify them as relative minima, maxima, or neither.
 - b. Over what interval(s) is $f(x)$ increasing? Decreasing?
 - c. Find the absolute minimum and absolute maximum $f(x)$ attains on the interval $[0, 8]$.
 - d. Find the inflection points. Over what intervals is $f(x)$ concave up? Concave down?
5. (20) Watsonovia Bank offers an account which pays interest at the rate of 6%, compounded continuously.
 - a. I deposit 10,000 dollars into an account. What is its value after 20 years?
 - b. Find the future value of a continuous money flow of 1,000 dollars per year for 40 years.
 - c. Find the accumulated present value of an investment over a 30 year period if there is a continuous money flow of 3,000 dollars per year.
6. (20) Consider $f(x, y) = x^2y^3 + xy + y$
 - a. Compute $f(1, 4)$.
 - b. Compute $f_y(x, y)$.
 - c. Compute $f_{yy}(x, y)$.
7. (20) Consider $f(x) = \frac{1}{x}$
 - a. Find the area of the region bounded by $f(x)$, the x -axis, $x = 1$ and $x = 2$.
 - b. Find the volume obtained by rotation the above region about the x -axis.
 - c. Compute $\int_1^{\infty} f(x) dx$.
8. (20) I wish to fence off a field. One side is bordered by a river. For the other three sides, I have 120 feet of fencing available.
 - a. Find the maximum area I can fence off.
 - b. Find the dimensions of the area (length and width).
9. (20)
 - a. Solve the variables-separable differential equation $\frac{dy}{dt} = -ky$.
 - b. Watsonium- \mathbb{X} decays at a rate of 4% per year. Find its half life.
 - c. Archaeologists who have found *The Milton Scrolls* have determined they contain 25% of their starting amount of Watsonium- \mathbb{X} . How old are the scrolls?