

# MA121 Elements of Calculus

## Exam 4 Form 42

24 November, 2008

*Instructions:* Show all work relevant to the solution of each problem. i.e. no credit will be given for “just the answers.” Please do *all* work in the Blue Books! There are **eight** problems which carry a total of 100 points. You will have until the end of class to complete this exam. Good luck!

(10 pts) **Problem 1.** Definitions and Concepts.

- Briefly explain the concepts of convergent and divergent integrals. How do they differ?
- Write a function,  $f$ , which satisfies the relationship  $\frac{df}{dx} = x^2$ .
- Write a function,  $f(x)$ , such that  $\int f(x)dx = \ln(x)$ .
- In the expression  $\lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i)\Delta x_i$ , what does  $n$  represent?
- Suppose the function  $v(t)$  gives the speed of a car at time  $t$ . What is described by the value  $\int_0^5 v(t)dt$ ?
- Identify the false statement amongst the three given.

- $\int kf(x)dx = k \int f(x)dx$
- $\int f(g(x))dx = \int f(x)dx \int g(x)dx$
- $\int f(x) + g(x)dx = \int f(x)dx + \int g(x)dx$

(30 pts) **Problem 2.** Compute each of the following:

- $\int 12x^3 + 4x dx$
- $\int \frac{4}{x} dx$
- $\int 3x^2(x^3 - 5)^{10} dx$
- $\int_0^1 12x^2 + 2x dx$

(10 pts) **Problem 3.** Find the area of the region bounded by the  $x$ -axis, the curve  $f(x) = 4x + 1$ ,  $x = 1$  and  $x = 3$ .

(10 pts) **Problem 4.** Find the volume of the solid constructed by rotating the above region about the  $x$ -axis.

(10 pts) **Problem 5.**

- Determine the anti-derivative of  $\frac{4x^3+9}{x^4+9x}$ .
- Use your answer from (a) to compute  $\int_1^{\infty} \frac{4x^3+9}{x^4+9x} dx$ . If you could not determine the anti-derivative, explain how you would solve the rest of the problem.

(10 pts) **Problem 6.**

- Find the point where the curves  $f(x) = x^3$  and  $g(x) = x^4$  intersect.
- Find the area of the region bounded by the curves  $f(x)$  and  $g(x)$ .

(10 pts) **Problem 7.** In 1988 the Milton's consumption of cat food was 350 metric tons per year. The amount consumed has increased at an average rate of 2% per year. Find the *total amount* of cat food consumed from 1988 to 2008.

(10 pts) **Problem 8.** Find the area under the piece-wise defined function from  $x = 0$  to  $x = 30$ .

$$f(x) = \begin{cases} 2x & \text{if } x \leq 10; \\ -2x + 40 & \text{if } x > 10. \end{cases}$$