

MA121 Elements of Calculus

Exam 4 Form 41

15 April, 2009

Instructions: Show all work relevant to the solution of each problem. i.e. no credit will be given for “just the answers.” Please do *all* work in the Blue Books! There are **eight** problems which carry a total of 106 points. You will have until the end of class to complete this exam. Good luck!

(10 pts) **Problem 1.** Definitions and Concepts.

- Write a function, $f(x)$, such that $\int f(x)dx = \ln(x)$.
- Write a function, f , which satisfies the relationship $\frac{df}{dx} = -2x$.
- Briefly explain the concepts of convergent and divergent integrals. How do they differ?
- In the expression $\lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i)\Delta x_i$, what does $f(x_i)$ represent?
- Suppose the function $A(t)$ gives the rate of oil consumption at year t . What is described by the value $\int_0^5 A(t)dt$?

(24 pts) **Problem 2.** Compute each of the following:

- $\int \frac{1}{x} - \sqrt{x} dx$
- $\int x^2 + \frac{3}{x} dx$
- $\int 20x^3(5x^4 - 100)^5 dx$

(12 pts) **Problem 3.** Find the area of the region bounded by the x -axis, the curve $f(x) = e^x$, $x = 0$ and $x = 1$.

(12 pts) **Problem 4.** Find the volume of the solid constructed by rotating the above region about the x -axis.

(12 pts) **Problem 5.** In the year 1900, Milton City switched from a coal to a petroleum-based economy. As such, the amount of coal used by Milton City in year t is modelled using the decreasing function $A(t) = 15000e^{-.05t}$.

- Find the anti-derivative of $A(t)$.
- Compute the total amount of coal used by Milton City from 1900 to indefinitely into the future.

(12 pts) **Problem 6.** Find the area of the region bounded by the curves $f(x) = x$ and $g(x) = x^2$.

(12 pts) **Problem 7.** Suppose I am saving for retirement by investing into an account which pays 1% interest, compounded continuously. There is a continuous flow of money into the account, at the rate of 4500 dollars per year. Find the total amount of money that will have accumulated in 40 years.

(12 pts) **Problem 8.** Find the area under the piece-wise defined function from $x = -10$ to $x = 10$.

$$f(x) = \begin{cases} -x & \text{if } x \leq 0; \\ x & \text{if } x > 0. \end{cases}$$