MA121 Elements of Calculus
28 March 2007
Exam 3 Review

(*) This is a review examination. The actual exam will not be as long. It is intended to help you review important concepts and give you an idea of *what kind* of questions may be asked (but *not* the questions themselves!).

Instructions: Show all work for full credit. Refer to any work done on separate pages.

1. Give an example of each of the following (You may present either a function of a graph of a function).
   - **a.** A function which satisfies the relationship $\frac{dP}{dt} = kP$.
   - **b.** A function which satisfies the relationship $\frac{dP}{dt} = -kP$.
   - **c.** A function which is its own derivative ($f(x) = f'(x)$).
   - **d.** A function on a closed interval, $[A, B]$, which has no relative mins or maxes, but has an absolute min and max
   - **e.** A function which has an absolute maximum
   - **f.** An increasing function with an absolute minimum
2. Differeniate.
   - **a.** $f(x) = \ln(3x)$
   - **b.** $f(x) = \log_4(3x^2)$
   - **c.** $f(x) = e^{3x^2+x}$
   - **d.** $f(x) = 5^{2x}$
3. Find the absolute maximum and minimum of $f(x) = x^2 + 3x$ on $[-4, 4]$.
4. Find the absolute maximum and minimum of $f(x) = \frac{x+3}{x}$ on $[-5, 5]$.
5. Find the absolute maximum and minimum of $f(x) = x^2 + \frac{250}{x}$
6. What is the function that satisfies $\frac{dP}{dt} = -.03P$?
7. What is the function that satisfies $P'(t) = .077P(t)$?
8. Watsonium-544 decays at a rate of 5.44% per year. Therefore, if $N(t)$ is the amount of Watsonium-544 at year t, then $\frac{dN}{dt} = -.0544N$.
   - **a.** Find the function $N(t)$ which satisfies the relationship and tells me how much Watsonium-544 I will have at time t if I initially had N_0 amount of Watsonium-544.
   - **b.** Find the half-life of Watsonium-544.
   - **c.** Archaeologists investigating the great Watson Pyramids stumble upon the Idol of Cy. It is known that Watsonium-544 decays into Watsonium-322, and this is used to determine that 88.5% of the initial amount of Watsonium-544 in the Idol of Cy has decayed. How old is the Idol of Cy?

9. Many savings accounts have notoriously low interest rates. I deposit \$400 into a savings account which pays an interest of 0.13%, compounded continuously. To pay for a new car, I will leave the money in the account until it grows to \$15,000. How long will I wait?

10. A box has a square base, and the sum of the length, width, and height must equal 60 inches. Depending upon how I set the length, width, and height within this constraint, the box will obtain varying volumes. Find the maximum volume I can obtain.

11. Find t such that $f(t) = 0$ if $f(t) = e^{t^2} - e$

12. Find the equation of the line tangent to $f(x)$ and through the point $(0, 5)$ if $f(x) = 5^{2x+1}$.

13. Find $f'(x)$ if $f(x) = \ln(\frac{3x^2+9}{x+3})$. Hint: This is easier if you review the properties of logarithms before diving in.

14. When Watson Tech sells computers at a price of \$1,000, the company averages about 185,000 sales. For each \$50 increase in price, however, 5,000 sales are lost. What is the domain of the profit function restricted to values which make sense in light of the application? What should Watson Tech sell their computers for to maximize profit?

15. In 2006, Watsonville ISD had a total population of 34,000 students. The population grows at an average rate of 1.5% per year. The school district will need to consider constructing a new school building when the total population reaches 36,000. Using the uninhibited growth model, predict the year when Watsonville ISD will need to have the funds required to build a new school.

16. Find the derivatives.

    a. $5^x x^5$

    b. $e^{x\ln(x)}$

    c. $x^{55}\log_5(x)$

    d. e^{x^2+4x+4}

17. Find $f(x)$ if $f'(x) = 3f(x)$.

18. Find the derivative of $f(x)$ if $f(x) = e^5$. What about if $f(x) = 2^5 3^5$?

19. I own a small house on the beach and I want to fence off a *rectangular* section of private beach along the oceanside. I have 180 feet of fencing I can use for the three edges, with the fourth edge of the lot being the water line (and hence, no need to be fenced). Find the maximum area I can enclose, and the lengths of all three edges which produce the maximum area.

20. Differentiate $f(x) = (e^{2x} + 50x)^5$

21. Integrate.

    a. $\int x^2 + 2x + 4dx$

    b. $\int \frac{15}{x}dx$

    c. $\int_0^1 3x^2 - 5xdx$

    d. $\int \sqrt{x} + \frac{1}{\sqrt{x}}dx.$

22. Find the area under the curve $f(x) = x^2 + 1$, bounded on the left by $x = 0$ and on the right by $x = 5$.

23. Find $f(x)$ such that $f'(x) = x - 6$ and $f(1) = 5$.