

MA121 Elements of Calculus

Exam 3 Form 32

March 25, 2009

Instructions: Show all work relevant to the solution of each problem. i.e. no credit will be given for “just the answers.” Please do *all* work in the Blue Books! There are **seven** problems which carry a total of 102 points. You will have until the end of class to complete this exam. Good luck!

(10 pts) **Problem 1.** Definitions and Concepts.

- Determine a function which satisfies the relationship $\frac{dP}{dt} = kP$.
- Give an example of a function whose derivative is $f'(x) = x^2$.
- True or False. The function $f(x) = 50e^{4x}$ has no absolute maximum or minimum on $(-\infty, \infty)$.
- Let the function $r(t)$ model *marginal revenue*, or the rate of change of revenue over time. What would the function $R(t)$, the anti-derivative of $r(t)$, model?
- Draw the graph of $f(x)$ which has **no** relative maximum, but does have an absolute maximum.

(20 pts) **Problem 2.** Differentiate.

- $f(x) = e^{5x^5}$
- $f(x) = \ln(2x)$
- $f(x) = \ln\left(\frac{x^2+5x+6}{x+2}\right)$.
- $f(x) = 5^x + \log_2(x^5)$

(20 pts) **Problem 3.** Let $f(x) = x^3 - 3x + 50$. Find the absolute maximum and minimum on the interval $[-5, 2]$.

(13 pts) **Problem 4.** Scientists unearth the legendary tomb of the Cat god Milton, lord of hairballs, and wish to determine its age using Carbon dating. They determine 31% of the tombs' original amount of Carbon-14 remains. Assuming Carbon-14 has a half-life of 5750 years, and the **exponential decay** relation $A(t) = A_0e^{-kt}$,

- Find the value of the unknown parameter k , where $A(t)$ is the current amount of Carbon-14 present, and t is the number of years that have passed.
- Determine the age of the tomb.

(13 pts) **Problem 5.** I wish to enclose a rectangular lot with fencing. The area of the lot must be 400 square feet. One side of the lot borders a river. Find the dimensions of the lot which minimize the amount of fencing required.

(13 pts) **Problem 6.** Milton City boasted a total population of 90,000 in 1990. In 2000, the population had increased to 100,000. Assuming exponential growth, predict the population in the year 2050.

(13 pts) **Problem 7.**

- Find the anti-derivative of $f(x) = x^2 + 3x + \sqrt{x}$.
- Find the anti-derivative of $g(x) = \frac{1}{x} + \frac{1}{x^4}$.