MA121 Elements of Calculus
Exam 3 Form 31

April 12, 2009

Instructions: Show all work relevant to the solution of each problem. i.e. no credit will be given for "just the answers." Please do *all* work in the Blue Books! There are **seven** problems which carry a total of 102 points. You will have until the end of class to complete this exam. Good luck!

(10 pts) **Problem 1.** Definitions and Concepts.

a. Determine a function which satisfies the relationship $\frac{dP}{dt} = kP$.

(*) $P(t) = Ce^{kt}$

b. Give an example of a function whose derivative is $f'(x) = x^2$.

(*) $f(x) = \frac{1}{3}x^3 + C$

c. True or False. The function $f(x) = 50e^{4x}$ has no absolute maximum or minimum on $(-\infty, \infty)$.

(*) True.

d. Let the function $r(t)$ model *marginal revenue*, or the rate of change of revenue over time. What would the function $R(t)$, the anti-derivative of $r(t)$, model?

(*) Revenue as a function of time.

e. Draw the graph of $f(x)$ which has **no** absolute maximum, but does have a relative maximum.

(*) One solution is to introduce a vertical asymptote into any of the examples we had in class.

(20 pts) **Problem 2.** Differentiate.

a. $f(x) = e^{2x^2}$

(*) $(4x)e^{2x^2}$

b. $f(x) = \ln(5x)$

(*) $\frac{5}{5x}$

c. $f(x) = \ln(\frac{x^2+5x+6}{x+3})$.

(*) $f(x) = \ln(x^2 + 5x + 6) - \ln(x + 3)$ Then $f'(x) = \frac{2x+5}{x^2+5x+6} - \frac{1}{x+3}$

d. $f(x) = 2^x + \log_5(x^2)$

(*) $\ln(2)2^x + \frac{1}{\ln(5)}\frac{2x}{x^2}$

(20 pts) **Problem** 3. Let $f(x) = x^3 - 3x + 50$. Find the absolute maximum and minimum on the interval $[-5, 2]$.

(*) $f'(x) = 3x^2 - 3$ Then solving $f'(x) = 3x^2 - 3 = 0$, I get $x^2 - 1 = 0$, or $x = \pm 1$. I will evaluate $f(x)$ for each of these values, as well as the closed end points of the interval. $f(-5) = -60$ $f(-1) = 52$ $f(1) = 48$ $f(2) = 52$ I have the min at $x = -5$. I have max at $x = -1$ and $x = 2$.

(13 pts) **Problem** 4. Scientists unearth the legendary tomb of the Cat god Milton, lord of hairballs, and wish to determine its age using Carbon dating. They determine 31% of the tombs' original amount of Carbon-14 remains. Assuming Carbon-14 has a half-life of 5750 years, and the **exponential decay** relation $A(t) = A_0 e^{-kt}$,

a. Find the value of the unknown parameter k, where $A(t)$ is the current amount of Carbon-14 present, and t is the number of years that have passed.

(*) I solve for k, $\frac{1}{2}A_0 = A_0 e^{-5750k}$ Dividing by A_0 I see $\frac{1}{2} = e^{-5750k}$ Applying $\ln()$ to both sides: $\ln(\frac{1}{2}) = -5750k$ Then $k = -\frac{\ln(\frac{1}{2})}{5750}$

b. Determine the age of the tomb.

(*) I have the model $A(t) = A_0 e^{\frac{\ln(\frac{1}{2})}{5750} t}$.

I want to find t when $A(t) = .31A_0$.

So I solve for t in $.31A_0 = A_0 e^{\frac{\ln(\frac{1}{2})}{5750} t}$.

Dividing by A_0: $.31 = e^{\frac{\ln(\frac{1}{2})}{5750} t}$.

Applying $\ln()$ to both sides: $\ln(.31) = \frac{\ln(\frac{1}{2})}{5750} t$.

Then dividing by $\frac{\ln(\frac{1}{2})}{5750}$ I see: $t = \frac{\ln(.31)}{\frac{\ln(\frac{1}{2})}{5750}}$.

(13 pts) **Problem 5.** I wish to enclose a rectangular lot with fencing. The area of the lot must be 400 square feet. One side of the lot borders a river. Find the dimensions of the lot which minimize the amount of fencing required.

(*) The amount of fencing required is given by $A = 2l + w$.

I have the constraint $lw = 400$.

Dividing by l I get $w = \frac{400}{l}$.

Then substituting back into my fencing function, I have A as a function of l:
$A(l) = 2l + \frac{400}{l}$.

Now $A'(l) = 2 - \frac{400}{l^2}$.

Solving $A'(l) = 2 - \frac{400}{l^2} = 0$, I first add the fraction to both sides: $2 = \frac{400}{l^2}$.

Then I can multiply by l^2: $2l^2 = 400$.

Dividing by 2: $l^2 = 200$.

Then $l = \sqrt{200}$.

I can plug l into the constraint to get $w = \frac{400}{\sqrt{200}}$.

(13 pts) **Problem 6.** Milton City boasted a total population of 90,000 in 1990. In 2000, the population had increased to 100,000. Assuming exponential growth, predict the population in the year 2050.

(*) The city population is given by the exponential model $P(t) = P_0 e^{kt}$.

If I let t be years since 1990, then I know first that $P_0 = 90000$.

So I have $P(t) = 90000e^{kt}$.

When $t = 10$ then $P(t) = 100000$, so $100000 = 90000e^{10k}$.

Then $\frac{10}{9} = e^{10k}$.

Taking $\ln()$ of both sides: $\ln(\frac{10}{9}) = 10k$.

Hence, $k = \frac{1}{10}\ln(\frac{10}{9})$.

Now I have $P(t) = 90000e^{\frac{1}{10}\ln(\frac{10}{9})t}$, so in 2050 the population is:

$P(60) = 90000e^{\frac{1}{10}\ln(\frac{10}{9})(60)}$.

(13 pts) **Problem 7.**

a. Find the anti-derivative of $f(x) = x^2 + 3x + \sqrt{x}$.

(*) $\frac{1}{3}x^3 + \frac{3}{2}x^2 + \frac{2}{3}x^{\frac{3}{2}} + C$

b. Find the anti-derivative of $g(x) = \frac{1}{x} + \frac{1}{x^4}$.

(*) $\ln(x) - \frac{1}{3}\frac{1}{x^3} + C$.