

Math 121 Elements of Calculus Exam 2

Here is a brief summary of what we've done:

1. We know we can find the average rate of change of a function between two points a and b via $\frac{f(b)-f(a)}{b-a}$. This gives the slope of the secant line.
2. We re-wrote the formula so instead of considering two independent points, it gives the average rate of change (or slope of secant line) between one point (x) , and another point a distance of h away $(x+h)$. So the new formula is $\frac{f(x+h)-f(x)}{h}$. This is the difference quotient.
3. We now want to find the instantaneous rate of change, or slope of the tangent line at x . This is essentially looking for the average rate of change between two points that occupy the same place, or letting the distance $h = 0$.
4. So we have a formula to find the slope of the tangent line by letting $h = 0$ in our difference quotient. But we can't just let $h = 0$ because we would be dividing by 0. But we can get around this by letting h approach 0 instead. So the slope of the tangent line at x is given by $\lim_{h \rightarrow 0} \frac{f(x+h)-f(x)}{h}$. We call this the derivative of a function, and denote it $f'(x)$.
5. This is a handy function because it gives us the (instantaneous) rate of change of a function at x . The potential for applications is enormous, so we invest a lot of effort into finding tricks to find the derivative in a quicker and easier manner than using the limit definition. The rules we've learned so far are
 - Sum/Difference/Constant Rule
 - Power Rule
 - Product Rule
 - Quotient Rule
 - Chain Rule
 - Extended Power Rule
6. Once we developed these tools, we just started talking about what we can do with them. We are interested in finding out where a function is increasing, decreasing, maximized, minimized, concave up/down, etc.

(*) This is a review examination. The actual exam will not be as long. It is intended to help you review important concepts and give you an idea of *what kind* of questions may be asked (but *not* the questions themselves!).

Instructions: Show all work for full credit. Please do *all* work in the Blue Books! Refer to any work done on separate pages. You will have 75 minutes to complete this exam.

1. (12) Definitions and Concepts: Please answer – out of – of each of the following.

a. Draw a graph of a function which is continuous, but not differentiable on its domain.

(*) Draw a graph with a “sharp turn” on the curve.

b. Draw a graph of a function which has a single critical point, but not a min or max.

(*) Look at the graph of $y = x^3$.

c. Draw a graph of a function $f(x)$ such that $f'(x) = 0$ for all x in its domain.

(*) Look at the graph of any constant function.

d. Give an equation of a function which is increasing for all x in its domain.

(*) Look at the graph of any increasing function, such as $y = x$.

e. Give an equation of a function which is decreasing for all x in its domain.

(*) For example, $y = -x$.

f. Give an equation of a function which is differentiable on its domain.

(*) Any polynomial, for instance.

g. Draw a graph of a function which is neither continuous nor differentiable on its domain.

(*) Draw a piecewise function with a “break”.

h. Is it possible to draw a graph which is not continuous, but differentiable on its domain?

(*) differentiability implies continuity, so no.

i. Draw a graph of a function such that the slope of line tangent to $f(x)$ is positive for all x in its domain.

(*) This is another way to prompt for an increasing function. See question (1d).

j. Draw a graph of a function which is concave down for all x in its domain.

(*) Look at the graph of $y = -x^2$.

2. (20) Please compute $f'(x)$ for – out of – of each of the following using the method prescribed. *SHOW ALL STEPS OF THE COMPUTATION!!*

a. $f(x) = 3x^2 + 2x$ using *power rule*.

$$(*) f'(x) = 6x + 2.$$

b. $f(x) = \sqrt{3}$ using *any rule*.

(*) $\sqrt{3}$ is a constant, and the derivative of any constant function is 0.

c. $f(x) = \sqrt{x}$ using *power rule*.

(*) Note that $\sqrt{x} = x^{\frac{1}{2}}$. I can use the power rule to differentiate this. The derivative is $f'(x) = \frac{1}{2}x^{-\frac{1}{2}}$.

d. $f(x) = (x - 1)(x - 2)$ using *power rule*.

(*) I have to expand the function (via FOIL) to arrive at a polynomial. Then I can use the power rule.

$$\begin{aligned} f(x) &= (x - 1)(x - 2) \\ &= x^2 - 3x + 2 \\ f'(x) &= 2x - 3 \end{aligned}$$

e. $f(x) = (x - 1)(x - 2)$ using *product rule*.

(*) I view $f(x)$ as the product of two functions $g(x)$ and $h(x)$ where $g(x) = x - 1$ and $h(x) = x - 2$. Then $g'(x) = 1$ and $h'(x) = 1$. The product rule states that the derivative of $g(x)h(x)$ is $g'(x)h(x) + g(x)h'(x)$ which is $1(x - 2) + 1(x - 1) = 2x - 3$.

f. $f(x) = x^{12}(x - 2)$ using *power rule*.

(*) First I need to make $f(x)$ look like a polynomial. So I distribute the x^{12} term.

$$\begin{aligned} f(x) &= x^{12}(x - 2) \\ &= x^{13} - 2x^{12} \\ f'(x) &= 13x^{12} - 24x^{11} \end{aligned}$$

g. $f(x) = x^{12}(x - 2)$ using *product rule*.

(*) Let $g(x) = x^{12}$ and $h(x) = x - 2$. Then $g'(x) = 12x^{11}$ and $h'(x) = 1$. Then we see the following.

$$\begin{aligned} f'(x) &= g'(x)h(x) + g(x)h'(x) \\ &= 12x^{11}(x - 2) + x^{12} \cdot 1 \\ &= 12x^{12} - 24x^{11} + x^{12} \\ &= 13x^{12} - 24x^{11} \end{aligned}$$

h. $f(x) = \frac{1}{x^3}$ using *power rule*.

$$(*) \frac{1}{x^3} = x^{-3}, \text{ so } f'(x) = -3x^{-4} = \frac{-3}{x^4}$$

i. $f(x) = \frac{x}{x^3}$ using *product rule*.

(*) $f(x) = x \cdot \frac{1}{x^3}$. I let $g(x) = x$ and $h(x) = \frac{1}{x^3}$. We already found $h'(x)$. $g'(x) = 1$.

$$\begin{aligned}f'(x) &= g'(x)h(x) + g(x)h'(x) \\ &= 1 \cdot \frac{1}{x^3} - x \cdot \frac{3}{x^4} \\ &= 1 \cdot \frac{1}{x^3} - \frac{3}{x^3} \\ &= \frac{-2}{x^3}\end{aligned}$$

j. $f(x) = \frac{1}{x^3}$ using *quotient rule*.

(*) The quotient rule lets us take the derivative of the quotient of two functions $\left(\frac{g(x)}{h(x)}\right)$. I consider $g(x) = 1$ and $h(x) = x^3$. Then $g'(x) = 0$ and $h'(x) = 3x^2$. From the quotient rule I can assemble these components correctly.

$$\begin{aligned}f'(x) &= \frac{h(x)g'(x) - g(x)h'(x)}{(h(x))^2} \\ &= \frac{x^3 \cdot 0 - 1 \cdot 3x^2}{(x^3)^2} \\ &= \frac{-3x^2}{x^6} \\ &= \frac{-3}{x^4}\end{aligned}$$

k. $f(x) = (x - 3)^2$ using *power rule*.

(*) I consider $f(x) = (x - 3)(x - 3)$. Then $f(x) = x^2 - 6x + 9$ using FOIL, and the derivative is obtained using the power rule. $f'(x) = 2x - 6$.

l. $f(x) = (x - 3)^2$ using *product rule*.

(*) I consider $f(x) = (x - 3)(x - 3)$, and also consider $g(x) = (x - 3)$ and $h(x) = (x - 3)$. Then $g'(x) = 1$ and $h'(x) = 1$. Using the product rule:

$$\begin{aligned}f'(x) &= g'(x)h(x) + g(x)h'(x) \\ &= 1(x - 3) + (x - 3)1 \\ &= 2(x - 3) \\ &= 2x - 6\end{aligned}$$

m. $f(x) = (x - 3)^2$ using *chain rule (or extended power rule)*.

(*) The extended power rule lets me treat the entire $(x - 3)$ term as “ x ” in the power rule. So the steps are the same... multiply the coefficient by the power, then drop the power by 1. There is also an added step, multiplying by the derivative of the contents of the parenthesis. But the derivative of $x - 3$ is 1. So $f'(x) = 2(x - 3)1 = 2(x - 3)$.

3. (12) Compute $f'(x)$ for each of the following using your favorite methods. Please indicate which method you are using.

a. $f(x) = (x^2 - 19)^{11}$.

(*) The extended power rule is the easiest route here. I multiply the coefficient of $(x^2 - 19)$ by 11, drop the power by 1, then multiply by the derivative of the contents of the parenthesis. This gives me

$$f'(x) = 11(x^2 - 19)^{10} 2x = 22x(x^2 - 19).$$

b. $f(x) = \frac{x^2+3}{x+3}$.

(*) I look for any ways to simplify this expression, but I will have to use the quotient rule here. I consider $g(x) = x^2 + 3$ and $h(x) = x + 3$. Then $g'(x) = 2x$ and $h'(x) = 1$.

$$\begin{aligned} f'(x) &= \frac{h(x)g'(x) - g(x)h'(x)}{(h(x))^2} \\ &= \frac{(x+3)2x - (x^2+3)1}{(x+3)^2} \\ &= \frac{2x^2 + 6x - x^2 - 3}{(x+3)^2} \\ &= \frac{x^2 + 6x - 3}{(x+3)^2} \end{aligned}$$

c. $f(x) = \sqrt{x^2 + 100}$.

(*) Whenever I see a square root I will re-write the expression. Note $f(x) = (x^2 + 100)^{\frac{1}{2}}$. It is then clear I should use the extended power rule. Multiply the coefficient of $(x^2 + 100)$ by the power, drop the power by 1, and multiply by the derivative of the contents of the parenthesis.

$$f'(x) = \frac{1}{2}(x^2 + 100)^{-\frac{1}{2}} 2x = (x^2 + 100)^{-\frac{1}{2}} = \frac{1}{\sqrt{x^2+100}}.$$

d. $f(x) = \frac{1}{\sqrt{x^2+100}}$.

(*) When I see a quotient it might be tempting to jump to the quotient rule, but I can avoid the long route by observing

$$f(x) = \frac{1}{(x^2+100)^{\frac{1}{2}}} = (x^2 + 100)^{-\frac{1}{2}}.$$

It is then clear I should use the extended power rule.

$$f'(x) = -\frac{1}{2}(x^2 + 100)^{-\frac{3}{2}} 2x = -(x^2 + 100)^{-\frac{3}{2}}.$$

e. $f(x) = (x^5 + x^3 + x)(x^4 + x^2 + 1)$.

(*) Instead of wasting time expanding this I will use the product rule. I consider $g(x) = x^5 + x^3 + x$ and $h(x) = x^4 + x^2 + 1$. Then $g'(x) = 5x^4 + 3x^2 + 1$ and $h'(x) = 4x^3 + 2x$.

$$\begin{aligned} f'(x) &= g'(x)h(x) + g(x)h'(x) \\ &= (5x^4 + 3x^2 + 1)(x^4 + x^2 + 1) + (x^5 + x^3 + x)(4x^3 + 2x) \end{aligned}$$

This is far enough.

f. Can you compute $\frac{d^2y}{dx^2}$ for $f(x) = x^9 + x^8 + x^7$? What about $\frac{d^3y}{dx^3}f(x)$

(*) $f'(x) = 9x^8 + 8x^7 + 7x^6$ using the power rule. To get the second derivative, I just take the derivative of the first derivative.
 $f''(x) = 72x^7 + 56x^6 + 42x^5$. To get the third derivative, I just take the derivative of the second derivative.
 $f'''(x) = 504x^6 + 336x^5 + 210x^4$.

4. (10) $f(x) = x^2 + 3x$.

a. Using the limit definition of a derivative, compute $f'(x)$.

(*)

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{(x+h)^2 + 3(x+h) - (x^2 + 3x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{x^2 + 2hx + h^2 + 3x + 3h - x^2 - 3x}{h} \\ &= \lim_{h \rightarrow 0} \frac{2hx + 3h + h^2}{h} \\ &= \lim_{h \rightarrow 0} 2x + 3 + h \\ &= 2x + 3 \end{aligned}$$

b. Find the equation of the line tangent to $f(x)$ at $x = 1$.

(*) A line has the form $y = mx + b$. We need to find m and b .

The slope of the line tangent to $f(x)$ at $x = 1$ is $f'(1) = 2(1) + 3 = 5$. This gives us $y = 5x + b$

To find b we plug in an (x, y) coordinate and solve for it. Since the line is tangent to the function at $x = 1$, the line passes through the point $(1, f(1)) = (1, 4)$. So we use that point. $4 = 5(1) + b \Rightarrow b = -1$. This gives $y = 5x - 1$ as the equation for the tangent line.

c. Find any critical points.

(*) Critical points are the values of x such that either $f'(x) = 0$ or where function is not differentiable. The function is a polynomial which is smooth and continuous everywhere, so there are no points where $f(x)$ is not differentiable. Solving $f'(x) = 0$ gives $2x + 3 = 0 \Rightarrow x = -\frac{3}{2}$. This is the only critical point.

d. On what interval(s) is $f(x)$ increasing?

(*) The critical point splits the real number line into two intervals, $(-\infty, -\frac{3}{2})$ and $(-\frac{3}{2}, \infty)$. I Evaluate $f'(x)$ at a point in the interval $(-\infty, -\frac{3}{2})$... say -2 . $f'(-2) = -1$, which is a negative number implies $f'(x) < 0$ for all x on that interval. Hence, $f(x)$ is decreasing for all x on the interval $(-\infty, -\frac{3}{2})$. For the other interval, I pick another point inside it. $f'(2) = 7$, which is positive, so $f(x)$ is increasing on $(-\frac{3}{2}, \infty)$.

5. (8) Find any critical points in the functions given.

a. $f(x) = x^2 - 9x + 2$

(*) I want to solve $f'(x) = 0$. From the power rule, $f'(x) = 2x - 9$, and $f'(x) = 0$ has one solution: $\frac{9}{2}$. So this is one critical point. The critical points can also be points where $f(x)$ is not differentiable. But since $f(x)$ is a polynomial and all polynomials are smooth and continuous, there are no such points. So $\frac{9}{2}$ is the only critical point.

b. $f(x) = \frac{2}{3}x^3 - 4x^2 - 4x + 99$

(*) Again, I want to solve $f'(x) = 0$. From the power rule, $f'(x) = 2x^2 - 8x - 4$. I will use the quadratic formula to pop the solutions out of the equation $f'(x) = 0$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{8 \pm \sqrt{64 - 4(2)(-4)}}{4} = \frac{8 \pm \sqrt{64 + 32}}{4} = 2 \pm \frac{\sqrt{96}}{4} = 2 \pm \sqrt{6}.$$

Since $f(x)$ is a polynomial, it is differentiable everywhere. So these are the only two critical points.

6. (16) Let $f'(x) = (x + 2)(x - 1)(x - 6)$ and $f''(x) = (x - 4)(3x + 2)$.

a. What are the critical points of $f(x)$?

(*) Solving $f'(x) = 0$, I get the critical points $-2, 1, 6$. Noting there are no points where $f(x)$ is not differentiable, there are no additional critical points.

b. On what intervals is $f(x)$ increasing?

(*) I have four intervals. $(-\infty, -2)$, $(-2, 1)$, $(1, 6)$, $(6, \infty)$. I'll pick a point inside each interval to evaluate $f'(x)$ at. If the result is negative, I know $f(x)$ is decreasing on that entire interval. If the result is positive, I know $f(x)$ is increasing at that interval.

0 is in the interval $(-2, 1)$ and $f'(0) = (2)(-1)(-6) = 6$, which is positive. So $f(x)$ is increasing on $(-2, 1)$.

10 is in the interval $(6, \infty)$ and $f'(10) = (12)(9)(4) = 416$, which is positive. So $f(x)$ is increasing on $(6, \infty)$.

c. On what intervals is $f(x)$ decreasing?

(*) -3 is in the interval $(-\infty, -2)$ and $f'(-3) = (-1)(-4)(-9) = -36$, which is negative. So $f(x)$ is decreasing on $(-\infty, -2)$.

2 is in the interval $(1, 6)$ and $f'(2) = (4)(1)(-4) = -16$, which is negative. So $f(x)$ is decreasing on $(1, 6)$.

d. On what points is the slope of the line tangent to $f(x)$ horizontal?

(*) This is equivalent to asking for what x is $f'(x) = 0$. The solutions we've obtained before are $-2, 1, 6$.

e. On what intervals is $f(x)$ concave up?

(*) The solutions to $f''(x) = 0$ give two inflection points: $-\frac{2}{3}, 4$. This gives us three intervals: $(-\infty, -\frac{2}{3})$, $(-\frac{2}{3}, 4)$ and $(4, \infty)$. I'll pick a point inside each interval to evaluate $f''(x)$ at. If the result is negative, $f(x)$ is concave down on that interval. If the result is positive, $f(x)$ is concave up on that interval. -2 is in the interval $(-\infty, -\frac{2}{3})$ and $f''(-2) = (-6)(-4) = 24$, which is positive. So $f(x)$ is concave up on $(-\infty, -\frac{2}{3})$. 5 is in the interval $(4, \infty)$ and $f''(5) = (1)(17) = 17$, which is positive. So $f(x)$ is concave up on $(4, \infty)$.

f. On what intervals is $f(x)$ concave down?

(*) 0 is in the interval $(-\frac{2}{3}, 4)$ and $f''(0) = (-4)(2) = -8$, which is negative. So $f(x)$ is concave down on $(-\frac{2}{3}, 4)$.

7. (10) During the course of its run, the temperature of a person who has caught Watsonitis can be modelled by the function $f(t) = 98.5 + .1(-t^2 + 14t + 1)$, where t is the number of days since having been introduced to the virus.

a. What is the maximum temperature the person will have?

(*) From the power rule, $f'(t) = -.2t + 1.4$. Solving $f'(t) = 0$, I get $t = 7$ as a critical point. Is it a min or a max? I can find out easily using the second derivative test.

From the power rule, $f''(t) = -.2$, so I evaluate at $t = 7$. $f''(7) = -.2$, which is negative. This means $f(t)$ is concave down around $t = 7$, so $t = 7$ is a relative maximum by the second derivative test.

The maximum point is $(7, f(7))$, which is $(7, 103.5)$. The maximum temperature is 103.5 , and it occurs at time $t = 7$.

b. Assume the virus dies when t reaches a value such that temperature returns to normal (98.6). Find this value of t .

(*) I want to solve the equation $f(t) = 98.6$. This doesn't require any calculus... it is a simple algebra problem.

$f(t) = 98.5 + .1(-t^2 + 14t + 1) = -.1t^2 + 1.4t + 98.6$. I want to solve $-.1t^2 + 1.4t + 98.6 = 98.6$.

$$-.1t^2 + 1.4t + 98.6 = 98.6$$

$$-.1t^2 + 1.4t = 0$$

$$t(-.1t + 1.4) = 0$$

$$t = 0 \quad -.1t + 1.4 = 0$$

$$t = 0 \quad .1t = 1.4$$

$$t = 0 \quad t = 14$$

The solution $t = 0$ refers to the start of the infection. The solution $t = 14$, therefore, refers to the end, which is what we are seeking.

c. For what values of t is temperature increasing?

(*) The lone critical point is $t = 7$, so the real number line is split into two intervals: $(-\infty, 7)$ and $(7, \infty)$. We've already shown the critical point is a maximum. A function is increasing before the maximum is obtained, so $f(x)$ must be increasing on the interval $(-\infty, 7)$.

d. For what values of t is temperature decreasing?

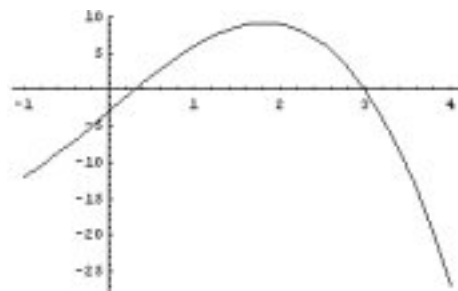
(*) A function is decreasing after the maximum is obtained, so $f(x)$ must be decreasing on the interval $(7, \infty)$.

e. Sketch a graph of $f(t)$ over the relevant points in the domain. Include any critical points, asymptotes, etc.

(*) Plot the graph on a calculator or computer and you will see a parabola that is concave down. We know it will be concave down because the lone critical point is a maximum. The max occurs at $t = 7$, and the roots of the parabola are at $t = 0$ and $t = 14$. There are no asymptotes because the function is a polynomial.

8. (12)

The graph of $f(x) = -x^3 + 10x - 3$ is given to the right.



a. Find the equation of the line tangent to $(3, 0)$.

(*) A line has the form $y = mx + b$. We need to find m and b .

The slope of the line tangent to $(3, 0)$ is given by $f'(3)$. $f'(x) = -3x^2 + 10$ by the power rule, so $f'(3) = -3(9) + 10 = -17$. So our partially completed line is $y = -17x + b$. Now to find b , all we need to do is to find a point to substitute for y and x , and solve for b . The tangent line goes through $(3, 0)$, so this is the only choice. $0 = -17(3) + b \Rightarrow b = 51$. So $y = -17x + 51$.

b. Use the first derivative to locate the exact location shown on the graph where the slope of the tangent line is 0.

(*) To find where the slope of the tangent line is 0, I solve $f'(x) = 0$.

$$\begin{aligned} f'(x) &= 0 \\ -3x^2 + 10 &= 0 \\ 3x^2 &= 10 \\ x^2 &= \frac{10}{3} \\ x &= \pm\sqrt{\frac{10}{3}} \end{aligned}$$

The location shown on the graph is $x = \sqrt{\frac{10}{3}}$.

c. Are there any other points not shown on the graph where the slope of the tangent line is 0?

(*) The location not shown on the graph is $x = -\sqrt{\frac{10}{3}}$.

- d. Use the second derivative to locate the intervals on which the graph is concave down.

(*) $f''(x) = -6x$. Solving $f''(x) = 0$, I get $x = 0$ as a solution. This means $x = 0$ is the location of an inflection point, where the concavity changes. So the domain is split into the intervals $(-\infty, 0)$, $(0, \infty)$.

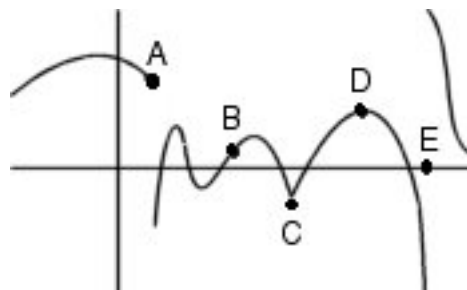
I pick a point inside the interval $(-\infty, 0)$, say, -1 . $f''(-1) = 6$, which is positive. This means $f(x)$ is concave up at $x = -1$. This means $f(x)$ must be concave up on the entire interval $(-\infty, 0)$.

I repeat the exercise for the interval $(0, \infty)$. $f''(1) = -6$ which is negative. So $f(x)$ is concave down at $x = 1$ and so must be concave down on the interval $(0, \infty)$. Look at the graph and you should be able to verify visually this is the correct answer.

- e. Sketch a graph of the first derivative.

(*) You will have to plot the curve $-3x^2 + 10$ into a calculator or computer.

The graph of $g(x)$ is given to the right.



f. At which points is $g(x)$ *not* continuous?

(*) A, E, because these are the places where the curve is “broken”.

g. At which points is $g(x)$ *not* differentiable?

(*) A, C, E. At C there is a sharp “point”. At A and E the function is discontinuous.

h. At which points is the slope of the line tangent to $g(x)$ horizontal?

(*) D is the only point. This can be verified because D appears to be a relative maximum.

i. Which points, if any, are the critical points?

(*) The critical points are the points where $g(x)$ is either not differentiable, or $g'(x) = 0$. So A, C, D, E are the points.

j. Which points, if any, are relative extrema?

(*) These are the points where $g'(x) = 0$ and it is either a relative min or max. D fits. Note this is *not* the same question as (h), because not every solution to $g'(x) = 0$ is a min or max. For example, $x = 0$ is a solution to $f'(x) = 0$ for $f(x) = x^3$. But $f(x) = x^3$ has neither a min or max!

k. Which points, if any, are relative maxima?

(*) Just the point D.

9. Consider $f(x) = \frac{x^2+6x+9}{3x+3}$.

a. What are the asymptotes of $f(x)$?

(*) Notice that the degree of the numerator is exactly one more than the degree of the denominator. So there could be an oblique asymptote. Let's check. I will use polynomial long division to do this. If you need to review how to perform this operation, Wikipedia has a good step-by-step explanation with examples. See:

http://en.wikipedia.org/wiki/Polynomial_long_division

I get $f(x) = \frac{1}{3}x + \frac{5}{3} + \frac{4}{3x+3}$. Because I expressed $f(x)$ as the sum of a linear equation (of the form $mx + b...$ in this case, $\frac{1}{3}x + \frac{5}{3}$), and a function $g(x)$ such that $\lim_{h \rightarrow \infty} g(x) = 0$, I have the oblique asymptote $y = \frac{1}{3}x + \frac{5}{3}$.

Also, I can solve the denominator for 0 to obtain the location of the vertical asymptote. Solving $3x + 3 = 0$, I get $x = -1$. I know this is a vertical asymptote and not a hole because if I could factor out the $(3x + 3)$ term, the remainder after polynomial division would have been 0 and not 4.

There are no horizontal asymptotes because the degree of the numerator is higher than the degree of the denominator.

b. Find $\lim_{x \rightarrow \infty} f(x)$

(*) $\lim_{x \rightarrow \infty} f(x) = \infty$ because the degree of the numerator is greater than the degree of the denominator.

10. Consider $f(x) = \frac{x^2+6x+9}{3x^2+5x+3}$.

a. What are the asymptotes of $f(x)$?

(*) I can't solve the denominator for 0. From the quadratic formula, the roots are $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-5 \pm \sqrt{25 - 36}}{2a}$. Notice $\sqrt{25 - 36} = \sqrt{-11}$ which has no real solution. So there can't be any vertical asymptotes because I can't find an x such that the denominator is 0.

The degree of the numerator and denominator are equal, which is one of the two cases for a horizontal asymptote. (The other is when the degree of the numerator is less than the degree of the denominator).

$\lim_{x \rightarrow \infty} f(x) = \frac{1}{3}$ since 1 is the coefficient of the dominant term of the numerator (x^2), and 3 is the coefficient of the dominant term of the denominator ($3x^2$). So there is a horizontal asymptote $y = \frac{1}{3}$.

Also, $\lim_{x \rightarrow -\infty} f(x) = \frac{1}{3}$. This yields the same horizontal asymptote as before.

There are no other asymptotes.

b. Find $\lim_{x \rightarrow \infty} f(x)$

(*) $\lim_{x \rightarrow \infty} f(x) = \frac{1}{3}$. I had to answer this question to find the horizontal asymptotes.