MA121 Elements of Calculus
17 February 2007
Exam 2 Form 20

> **(*)** This is a review examination. The actual exam will not be as long. It is intended to help you review important concepts and give you an idea of *what kind* of questions may be asked (but *not* the questions themselves!).

Instructions: Show all work for full credit. Please do *all* work in the Blue Books! Refer to any work done on separate pages. You will have 75 minutes to complete this exam.

1. (12) Definitions and Concepts: Please answer – out of – of each of the following.

   a. Draw a graph of a function which is continuous, but not differentiable on its domain.

   b. Draw a graph of a function which has a single critical point, but not a min or max.

   c. Draw a graph of a function $f(x)$ such that $f'(x) = 0$ for all x in its domain.

   d. Give an equation of a function which is increasing for all x in its domain.

   e. Give an equation of a function which is decreasing for all x in its domain.

   f. Give an equation of a function which is differentiable on its domain.

   g. Draw a graph of a function which is neither continuous nor differentiable on its domain.

   h. Is it possible to draw a graph which is not continuous, but differentiable on its domain?

   i. Draw a graph of a function such that the slope of line tangent to $f(x)$ is positive for all x in its domain.

   j. Draw a graph of a function which is concave down for all x in its domain.

2. (20) Please compute $f'(x)$ for – out of – of each of the following using the method prescribed. *SHOW ALL STEPS OF THE COMPUTATION!!*

   a. $f(x) = 3x^2 + 2x$ using *power rule*.

   b. $f(x) = \sqrt{3}$ using *any rule*.

   c. $f(x) = \sqrt{x}$ using *power rule*.

   d. $f(x) = (x-1)(x-2)$ using *power rule*.

   e. $f(x) = (x-1)(x-2)$ using *product rule*.

   f. $f(x) = x^{12}(x-2)$ using *power rule*.

   g. $f(x) = x^{12}(x-2)$ using *product rule*.

   h. $f(x) = \frac{1}{x^3}$ using *power rule*.

   i. $f(x) = \frac{x}{x^3}$ using *product rule*.

   j. $f(x) = \frac{1}{x^3}$ using *quotient rule*.

   k. $f(x) = (x-3)^2$ using *power rule*.

   l. $f(x) = (x-3)^2$ using *product rule*.

   m. $f(x) = (x-3)^2$ using *chain rule (or extended power rule)*.

3. (12) Compute $f'(x)$ for each of the following using your favorite methods. Please indicate which method you are using.

   a. $f(x) = (x^2 - 19)^{11}$.

   b. $f(x) = \frac{x^2+3}{x+3}$.

   c. $f(x) = \sqrt{x^2 + 100}$.

   d. $f(x) = \frac{1}{\sqrt{x^2+100}}$.

   e. $f(x) = (x^5 + x^3 + x)(x^4 + x^2 + 1)$.

   f. Can you compute $\frac{d^2y}{dx^2}$ for $f(x) = x^9 + x^8 + x^7$? What about $\frac{d^3y}{dx^3}f(x)$

4. (10) $f(x) = x^2 + 3x$.

   a. *Using the limit definition of a derivative*, compute $f'(x)$.

   b. Find the equation of the line tangent to $f(x)$ at $x = 1$.

   c. Find any critical points.

   d. On what interval(s) is $f(x)$ increasing?

5. (8) Find any critical points in the functions given.

   a. $f(x) = x^2 - 9x + 2$

   b. $f(x) = \frac{2}{3}x^3 - 4x^2 - 4x + 99$

6. (16) Let $f'(x) = (x + 2)(x - 1)(x - 6)$ and $f''(x) = (x - 4)(3x + 2)$.

   a. What are the critical points of $f(x)$?

   b. On what intervals is $f(x)$ increasing?

   c. On what intervals is $f(x)$ decreasing?

   d. On what points is the slope of the line tangent to $f(x)$ horizontal?

   e. On what intervals is $f(x)$ concave up?

   f. On what intervals is $f(x)$ concave down?

7. (10) During the course of its run, the temperature of a person who has caught Watsonitis can be modelled by the function $f(t) = 98.5 + .1(-t^2 + 14t + 1)$, where t is the number of days since having been introduced to the virus.

   a. What is the maximum temperature the person will have?

   b. Assume the virus dies when t reaches a value such that temperature returns to normal (98.6). Find this value of t.

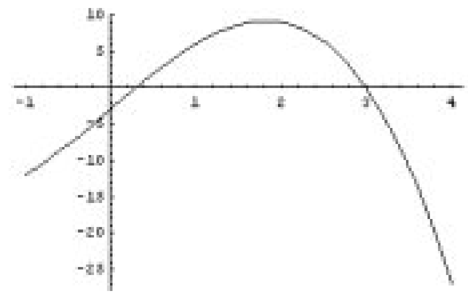
   c. For what values of t is temperature increasing?

   d. For what values of t is temperature decreasing?

   e. Sketch a graph of $f(t)$ over the relevant points in the domain. Include any critical points, asymptotes, etc.

8. (12)

The graph of $f(x) = -x^3 + 10x - 3$ is given to the right.



a. Find the equation of the line tangent to $(3, 0)$.

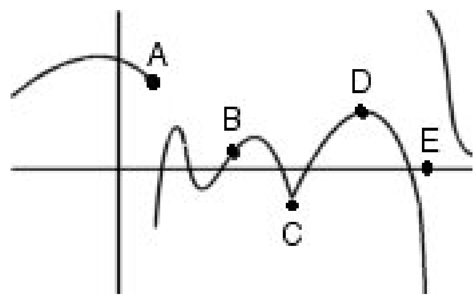
b. Use the first derivative to locate the exact location shown on the graph where the slope of the tangent line is 0.

c. Are there any other points not shown on the graph where the slope of the tangent line is 0?

d. Use the second derivative to locate the intervals on which the graph is concave down.

e. Sketch a graph of the first derivative.

The graph of $g(x)$ is given to the right.



f. At which points is $g(x)$ *not* continuous?

g. At which points is $g(x)$ *not* differentiable?

h. At which points is the slope of the line tangent to $g(x)$ horizontal?

i. Which points, if any, are the critical points?

j. Which points, if any, are relative extrema?

k. Which points, if any, are relative maxima?

9. Consider $f(x) = \frac{x^2+6x+9}{3x+3}$.

a. What are the asymptotes of $f(x)$?

b. Find $\lim_{x\to\infty} f(x)$

10. Consider $f(x) = \frac{x^2+6x+9}{3x^2+5x+3}$.

a. What are the asymptotes of $f(x)$?

b. Find $\lim_{x\to\infty} f(x)$