MA121 Elements of Calculus
Exam 2 Form 22 Solution

23 February, 2009

Instructions: Show all work relevant to the solution of each problem. i.e. no credit will be given for "just the answers." Please do *all* work in the Blue Books! There are **eight** problems which carry a total of 104 points. You will have until the end of class to complete this exam. Good luck!

(15 pts) **Problem 1.** Definitions and Concepts.

a. True or False. If $f(x)$ is continuous at every point x on its domain, I can compute $f'(x)$ for every x.

(*) False. As a counter-example, consider a sharp point.

b. True or False. A function can be both concave down and increasing on its domain.

(*) True.

c. Draw the graph of a function which has no critical points.

(*) One solution is any straight line.

d. Draw the graph of a function which satisfies the condition $f'(x) \leq 0$ for all x.

(*) Any function which never increases.

e. Which of the three statements below is **false**? (Multiple answers possible).

I. $\frac{d}{dx}(f(x)+g(x)) = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$.

II. $\frac{d}{dx}(f(x)g(x)) = \frac{d}{dx}f(x)\frac{d}{dx}g(x)$.

III. $\frac{d}{dx}(af(x)) = a\frac{d}{dx}f(x)$, where a is constant.

(*) II is false.

f. Which of the three statements below is **false**? (Multiple answers possible).

I. If $f(x) = k$ for some constant k, then $f'(x) = 0$.

II. If $f'(x) > 0$ then $f(x) > 0$.

III. If $f''(x) > 0$ then $f'(x) > 0$.

(*) II and III are false.

(10 pts) **Problem 2.** Using **the limit definition of the derivative**, find $f'(x)$ where $f(x) = x^2 - 1$.

(*)

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \\ &= \lim_{h\to 0} \frac{(x+h)^2-1-(x^2-1)}{h} \\ &= \lim_{h\to 0} \frac{x^2+2hx+h^2-1-x^2+1}{h} \\ &= \lim_{h\to 0} \frac{2hx+h^2}{h} \\ &= \lim_{h\to 0} 2x+h \\ &= 2x \end{aligned}$$

(15 pts) **Problem 3.** Compute $f'(x)$ using the **method prescribed**. Show all steps in your work.

a. $f(x) = 4x^3 + 4x^2 + 99$ using **power rule**.

(*) $12x^2 + 8x$

b. $f(x) = x^{21}(x^2 - 1)$ using **product rule**.

(*) $21x^{20}(x^2 - 1) + x^{21}(2x)$

c. $f(x) = \frac{x-1}{x+1}$ using **quotient rule**.

(*) $\frac{(x+1)(1)-(x-1)(1)}{(x+1)^2}$

(10 pts) **Problem** 4. Compute $f'(x)$ using any means. Show all steps in your work.

a. $f(x) = (9x^3 + 3)^9$.

(*) $9(9x^3 + 3)^8(27x^2)$

b. $f(x) = \frac{1}{\sqrt{x^9+3}}$.

(*) $-\frac{1}{2}(x^9 + 3)^{-\frac{3}{2}}(9x^8)$

(10 pts) **Problem** 5. Determine $f''(x)$ (the second derivative) of $f(x)$ where $f(x) = (x + 1)^8$.

(*) $f'(x) = 8(x + 1)^7$

$f''(x) = 56(x + 1)^6$

(10 pts) **Problem** 6. If $f(x) = x^3 + 1$, Find the *equation* of the tangent line at $x = 2$.

(*) $f'(x) = 3x^2$.

$m = f'(2) = 12$.

$y = 12x + b$.

When $x = 2$, then $y = f(2) = 9$.

$9 = 12(2) + b$

$-15 = b$

$y = 12x - 15$.

(20 pts) **Problem** 7. Let $f(x) = x^3 - 12x + 16$.

a. Locate the critical points of $f(x)$.

(*) $f'(x) = 3x^2 - 12$

Setting $f'(x) = 0$, I get:

$$\begin{aligned} 3x^2 - 12 &= 0 \\ 3x^2 &= 12 \\ x^2 &= 4 \\ x &= \pm 2 \end{aligned}$$

b. On what interval(s) is $f(x)$ increasing?

(*) I divide the real number line into three intervals, with the divisions being the critical points.
For $(-\infty, -2)$, $f'(-5) > 0$. Hence, $f(x)$ is increasing on this interval.
For $(-2, 2)$, $f'(0) < 0$. Hence, $f(x)$ is decreasing on this interval.
For $(2, \infty)$, $f'(5) > 0$. Hence, $f(x)$ is increasing on this interval.

c. Locate the inflection points of $f(x)$.

(*) $f''(x) = 6x$.
Solve $f''(x) = 0$ to get $x = 0$.

d. On what interval(s) is $f(x)$ concave up?

(*) To the right of $x = 0$: $f''(-5) < 0$. Hence, $f(x)$ is concave down on the interval $(-\infty, 0)$.
To the left of $x = 0$: $f''(5) > 0$. Hence, $f(x)$ is concave up on the interval $(0, \infty)$.

(14 pts) **Problem** 8. The graph of $g(x)$ is given below.

a. At which points is $g(x)$ *not* continuous?

(*) B

b. At which points is $g(x)$ *not* differentiable?

(*) B, D

c. At which points is the slope of the line tangent to $g(x)$ horizontal?

(*) A, C, E, F, G

d. Which points, if any, are the critical points?

(*) A, B, C, D, E, F, G

e. Which points, if any, are relative extrema?

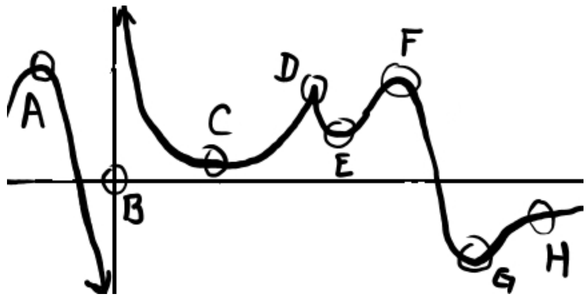
(*) A, C, E, F, G

f. Which points, if any, are relative maxima?

(*) A, F

g. Which point, if any, is the relative minima?

(*) C, E, G


A
B
C
D
E
F
G
H