

# MA121 Elements of Calculus

## Exam 2 Form 21

23 February, 2009

*Instructions:* Show all work relevant to the solution of each problem. i.e. no credit will be given for “just the answers.” Please do *all* work in the Blue Books! There are **eight** problems which carry a total of 104 points. You will have until the end of class to complete this exam. Good luck!

(15 pts) **Problem 1.** Definitions and Concepts.

- True or False. If  $f(x)$  is continuous at every point  $x$  on its domain, I can compute  $f'(x)$  for every  $x$ .
- True or False. A function can be both concave down and increasing on its domain.
- Draw the graph of a function which has no critical points.
- Draw the graph of a function which satisfies the condition  $f'(x) \leq 0$  for all  $x$ .
- Which of the three statements below is **false**? (Multiple answers possible).

I.  $\frac{d}{dx}(f(x) + g(x)) = \frac{d}{dx}f(x) + \frac{d}{dx}g(x).$

II.  $\frac{d}{dx}(f(x)g(x)) = \frac{d}{dx}f(x)\frac{d}{dx}g(x).$

III.  $\frac{d}{dx}(af(x)) = a\frac{d}{dx}f(x)$ , where  $a$  is constant.

- f. Which of the three statements below is **false**? (Multiple answers possible).

I. If  $f(x) = k$  for some constant  $k$ , then  $f'(x) = 0$ .

II. If  $f'(x) > 0$  then  $f(x) > 0$ .

III. If  $f''(x) > 0$  then  $f'(x) > 0$ .

(10 pts) **Problem 2.** Using **the limit definition of the derivative**, find  $f'(x)$  where  $f(x) = x^2 + 1$ .

(15 pts) **Problem 3.** Compute  $f'(x)$  using the **method prescribed**. Show all steps in your work.

a.  $f(x) = 3x^4 + 2x^2 + 99$  using **power rule**.

b.  $f(x) = x^{20}(x^2 + 1)$  using **product rule**.

c.  $f(x) = \frac{x+1}{x-1}$  using **quotient rule**.

(10 pts) **Problem 4.** Compute  $f'(x)$  using any means. Show all steps in your work.

a.  $f(x) = (10x^3 + 3)^9$ .

b.  $f(x) = \frac{1}{\sqrt{x^3+9}}$ .

(10 pts) **Problem 5.** Determine  $f''(x)$  (the second derivative) of  $f(x)$  where  $f(x) = (x + 1)^8$ .

(10 pts) **Problem 6.** If  $f(x) = x^3 + 1$ , Find the *equation* of the tangent line at  $x = 2$ .

(20 pts) **Problem 7.** Let  $f(x) = x^3 - 12x + 16$ .

- Locate the critical points of  $f(x)$ .
- On what interval(s) is  $f(x)$  increasing?
- Locate the inflection points of  $f(x)$ .
- On what interval(s) is  $f(x)$  concave up?

(14 pts) **Problem 8.** The graph of  $g(x)$  is given below.

- At which points is  $g(x)$  *not* continuous?
- At which points is  $g(x)$  *not* differentiable?
- At which points is the slope of the line tangent to  $g(x)$  horizontal?
- Which points, if any, are the critical points?
- Which points, if any, are relative extrema?
- Which points, if any, are relative maxima?
- Which point, if any, is the relative minima?

