

MA121 Elements of Calculus

Exam 1 Review Questions

September 6, 2008

1. Definitions and Concepts.

- Give an example of an equation in which the value of y varies inversely with the value of x .
- Give an example of a function whose domain is $\{x|x \neq 4\}$.
- Give an example of a function whose domain is $\{x|x \geq 4\}$.
- Give an example of a rational function which has a vertical asymptote at $x = 3$.
- Give an example of a graph of a function which is continuous on the interval $[0, 5]$.
- What is the difference between continuity at a point and continuity over an interval?
- If the limit as x approaches c from the left is the same as the limit as x approaches c from the right, what can I say about the general limit?
- True or False. If true, give a short statement as to why. If false, provide a counter-example. If a function is continuous at $x = 0$, then it must be continuous on the interval $(-1, 1)$.

2. a. Simplify $\frac{(x^2y^3)^2}{(x^3y^2)^3}$

b. Simplify $(x^{-2}y^{-2})^3(x^2y^3)^2$

d. Express $(x - 3)(x + 2)$ as a quadratic of the form $Ax^2 + Bx + C$. What is the degree of the resulting polynomial? What are the x and y intercepts?

e. Simplify $\frac{x^2+5x+6}{x^2+7x+12}$. What is the domain of the function? What are the locations of any asymptotes or holes?

3. Consider the function $f(x) = x^2 - 1$.

a. Compute $f(-2)$.

b. Compute the difference quotient, $\frac{f(x+h)-f(x)}{h}$.

c. What is the average rate of change as x changes from 2 to 7?

d. Compute $\lim_{x \rightarrow 0} f(x)$. Is $f(x)$ continuous at $x = 0$? Why or why not?

4. Find the equilibrium point for the given demand and supply functions. Consider only values which make sense with respect to the application.

$$\begin{aligned}D(p) &= -p + 13 \\S(p) &= p^2 - p - 3\end{aligned}$$

5. Consider the piecewise function defined below.

$$f(x) = \begin{cases} x + 2 & \text{if } x \neq 4; \\ 8 & \text{if } x = 4. \end{cases}$$

- a. Evaluate $f(4)$.
- b. Evaluate $\lim_{x \rightarrow 4} f(x)$.
- c. Is $f(x)$ continuous at $x = 3$? What about $x = 4$?
- d. Is $f(x)$ continuous on the interval $[0, 10]$? Why or why not?

6. Evaluate.

- a. $\lim_{x \rightarrow 1^-} \frac{1}{x-1}$
- b. $\lim_{x \rightarrow 1} \frac{1}{x-1}$
- c. $\lim_{x \rightarrow \infty} \frac{1}{x-1}$
- d. $\lim_{x \rightarrow 4^+} \frac{x^2-16}{x-4}$
- e. $\lim_{h \rightarrow 0} hx^2 + h^2x + x + 3$
- f. $\lim_{x \rightarrow \infty} \frac{5x^2-19x+2}{x^2-3}$

7. Watson Enterprises is considering producing and selling cat-proof keyboards. They plan to sell each keyboard for 40 dollars. Materials for each keyboard cost 20 dollars, and the machines to produce the keyboards cost 100,000 dollars. The machines must only be purchased once.

- a. Set up the linear function $C(x)$ which gives the total cost of producing x keyboards.

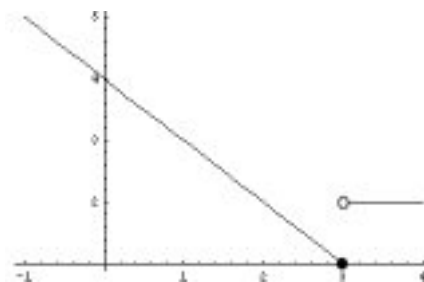
The cost per unit is the cost of the materials plus an equal fraction of the cost of the machine. So, for instance, if five keyboards are produced, the cost per keyboard is 20 dollars plus one-fifth of the cost of the machine (20,000 dollars). So the cost per unit if x units are produced is given by

$$M(x) = \frac{20x+100000}{x}$$

- b. What is the cost per unit if 10 keyboards are produced?
- c. Compute $\lim_{x \rightarrow \infty} M(x)$.
- d. What does your answer to (c) tell us about the cost per unit?

8. The graph of $f(x)$ is given to the right.

- a. Evaluate $\lim_{x \rightarrow 3^-} f(x)$.
- b. Evaluate $\lim_{x \rightarrow 3^+} f(x)$.
- c. Evaluate $\lim_{x \rightarrow 3} f(x)$.
- d. Is $f(x)$ continuous at $x = 3$?
- e. Is $f(x)$ continuous on the interval $[-1, 2]$?



The graph of $g(x)$ is given to the right.

- f. Evaluate $\lim_{x \rightarrow 0^-} g(x)$.
- g. Evaluate $\lim_{x \rightarrow 0^+} g(x)$.
- h. Is $g(x)$ continuous on the interval $[-1, 1]$?

