

# MA121 Elements of Calculus

## Exam 1 Form 12

February 15, 2009

*Instructions:* Show all work relevant to the solution of each problem. i.e. no credit will be given for “just the answers.” Please do *all* work in the Blue Books! There are **six** problems which carry a total 100 points. You will have until the end of class to complete this exam. Good luck!

(10 pts) **Problem 1.** Definitions and concepts.

- a. If the limit as  $x$  approaches  $c$  from the left is the same as the limit as  $x$  approaches  $c$  from the right, what can I say about the general limit?

(\*) It exists, and is equal to both “sided” limits.

- b. True or false. If the function  $f(x)$  is continuous on the interval  $(-5, 3)$ , then  $f(x)$  is continuous at the point  $x = 2$ .

(\*) True.

- c. Draw the graph of a function which is not continuous at the point  $x = 3$ .

(\*) Place a hole or vertical asymptote at  $x = 3$ .

- d. Draw the graph of a function for which the limit as  $x$  approaches 2 does not exist.

(\*) Place a vertical asymptote or “sharp turn” at  $x = 2$ .

- e. True or false. Consider the function  $g(x)$ . Suppose the average rate of change from  $A$  to  $B$  is positive. Then the slope of the secant line from  $A$  to  $B$  must be positive as well.

(\*) True.

(20 pts) **Problem 2.** Consider the rational function  $f(x) = \frac{x+9}{x^2-81}$ .

- a. What is the domain of  $f(x)$ ?

(\*) Solve  $x^2 - 81 = 0$  to get  $x = \pm 9$ .  
Domain is  $\{x \mid x \neq \pm 9\}$ .

- b. Compute  $\lim_{x \rightarrow 9} f(x)$ .

(\*)  $\frac{x+9}{x^2-81} = \frac{1}{x-9}$   
It is still not possible to plug in  $x = 9$  and get a real number.  
Hence, the limit does not exist.

c. Compute  $\lim_{x \rightarrow \infty} f(x)$ .

(\*) The degree of the numerator is less than the degree of the denominator. So the limit is 0.

(20 pts) **Problem 3.** Consider the polynomial function  $p(x) = x^2 + 8$ .

a. Compute  $p(x+h)$ .

(\*)  $p(x+h) = (x+h)^2 - 8 = x^2 + 2hx + h^2 - 8$ .

b. Find a simplified form of the difference quotient,  $\frac{p(x+h)-p(x)}{h}$ .

(\*)  $\frac{p(x+h)-p(x)}{h} = \frac{x^2+2hx+h^2-8-x^2+8}{h} = \frac{2hx+h^2}{h} = 2x+h$

c. Determine the average rate of change of  $p(x)$  from  $x = -2$  to  $x = 2$ .

(\*)  $\frac{p(2)-p(-2)}{2-(-2)} = \frac{12-12}{4} = 0$ .

d. Write the equation of the secant line from  $x = -2$  to  $x = 2$ .

(\*) The slope of the secant line is 0.  
I have  $y = 0x + b$ .  
Plug in  $x = 2, y = p(2) = 12$  to get  $12 = b$ .  
 $y = 12$ .

(20 pts) **Problem 4.** Consider the piecewise function defined below.

$$f(x) = \begin{cases} x^3 + 1 & \text{if } x < 0; \\ x^2 + x & \text{if } x \geq 0. \end{cases}$$

a. Evaluate  $f(0)$ .

(\*)  $f(0) = 0$

b. Evaluate  $\lim_{x \rightarrow 0^-} f(x)$ .

(\*) 1

c. Evaluate  $\lim_{x \rightarrow 0} f(x)$ .

(\*) Does not exist (the right-hand limit is 0).

d. Is  $f(x)$  continuous at  $x = 0$ ?

(\*) No. The limit does not exist.

e. Is  $f(x)$  continuous on the interval  $(-2, 2)$ ? Why or why not?

(\*) No. Because it is not continuous at every point in the interval.

(10 pts) **Problem 5.** Suppose I am producing cats. When I charge 20 dollars for a bag, I sell 30 units. When I charge 40 dollars for a bag, I sell 15 units.

- a. Write down the *slope* and *y-intercept* of the linear *demand equation*,  $D(p)$ , which gives the number of guests as a function of the price,  $p$ .

(\*) Units sold  $y$  is a function of price  $p$ .

$$\text{Slope is } \frac{y_2 - y_1}{p_2 - p_1} = \frac{40 - 20}{15 - 20} = -4.$$

$$D(p) = mp + b. \quad m = -4.$$

$$D(p) = -4p + b. \quad \text{When } p = 20 \text{ then } D(p) = 30.$$

$$30 = -4(20) + b$$

$$b = 110.$$

$$D(p) = -4p + 110.$$

- b. The *supply equation*,  $S(p)$ , gives the number of units I am willing to rent out as a function of the price,  $p$ . Suppose  $S(p)$  is defined as follows:

$$S(p) = 50 - p$$

What is the equilibrium price (The price for which supply matches demand)? Consider only values which make sense with respect to the context of the application.

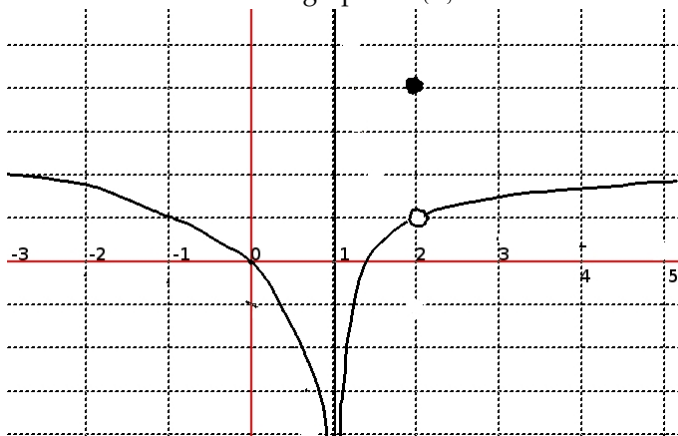
(\*)  $D(p) = S(p)$

$$-4p + 110 = 50 - p$$

$$3p = 60$$

$$p = 20.$$

(20 pts) **Problem 6.** Consider the graph of  $f(x)$  as shown below.



- a. Evaluate  $\lim_{x \rightarrow 1^-} f(x)$ .

(\*) DNE or  $-\infty$ .

b. Evaluate  $\lim_{x \rightarrow 1^+} f(x)$ .

(\*) DNE or  $-\infty$ .

c. Evaluate  $\lim_{x \rightarrow 1} f(x)$ .

(\*) DNE or  $-\infty$ .

d. Is  $f(x)$  continuous at  $x = 1$  Why or why not?

(\*) No.  $f(1)$  is not a real number.

e. Evaluate  $\lim_{x \rightarrow 2} f(x)$ .

(\*) 1

f. Evaluate  $f(2)$ .

(\*) 4

g. Is  $f(x)$  continuous at  $x = 2$  Why or why not?

(\*) No. The limit does not match  $f(2)$ .

h. Is  $f(x)$  continuous on the interval  $(-3, 0]$ ? Why or why not?

(\*) Yes.  $f(x)$  is continuous at every point in the interval.