

MA121 Elements of Calculus

Exam 1 Form 12

January 28, 2009

Instructions: Show all work relevant to the solution of each problem. i.e. no credit will be given for “just the answers.” Please do *all* work in the Blue Books! There are **six** problems which carry a total 100 points. You will have until the end of class to complete this exam. Good luck!

(10 pts) **Problem 1.** Definitions and concepts.

- If the limit as x approaches c from the left is the same as the limit as x approaches c from the right, what can I say about the general limit?
- True or false. If the function $f(x)$ is continuous on the interval $(-5, 3)$, then $f(x)$ is continuous at the point $x = 2$.
- Draw the graph of a function which is not continuous at the point $x = 3$.
- Draw the graph of a function for which the limit as x approaches 2 does not exist.
- True or false. Consider the function $g(x)$. Suppose the average rate of change from A to B is positive. Then the slope of the secant line from A to B must be positive as well.

(20 pts) **Problem 2.** Consider the rational function $f(x) = \frac{x+9}{x^2-81}$.

- What is the domain of $f(x)$?
- Compute $\lim_{x \rightarrow 9} f(x)$.
- Compute $\lim_{x \rightarrow \infty} f(x)$.

(20 pts) **Problem 3.** Consider the polynomial function $p(x) = x^2 + 8$.

- Compute $p(x+h)$.
- Find a simplified form of the difference quotient, $\frac{p(x+h)-p(x)}{h}$.
- Determine the average rate of change of $p(x)$ from $x = -2$ to $x = 2$.
- Write the equation of the secant line from $x = -2$ to $x = 2$.

(20 pts) **Problem 4.** Consider the piecewise function defined below.

$$f(x) = \begin{cases} x^3 + 1 & \text{if } x < 0; \\ x^2 + x & \text{if } x \geq 0. \end{cases}$$

- Evaluate $f(0)$.
- Evaluate $\lim_{x \rightarrow 0^-} f(x)$.
- Evaluate $\lim_{x \rightarrow 0} f(x)$.
- Is $f(x)$ continuous at $x = 0$?
- Is $f(x)$ continuous on the interval $(-2, 2)$? Why or why not?

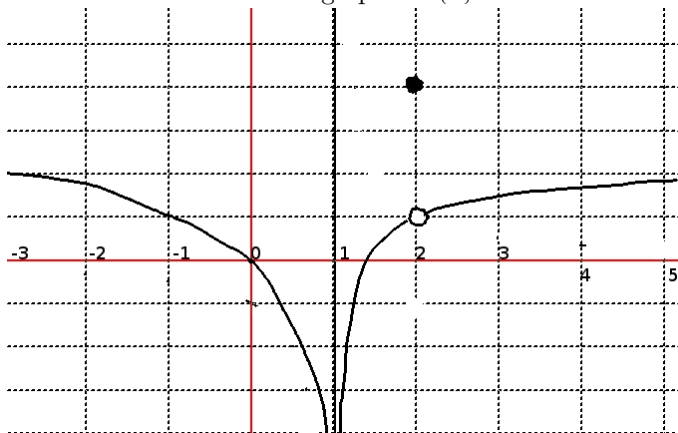
(10 pts) **Problem 5.** Suppose I am producing cats. When I charge 20 dollars for a bag, I sell 30 units. When I charge 40 dollars for a bag, I sell 15 units.

- Write down the *slope* and *y-intercept* of the linear *demand equation*, $D(p)$, which gives the number of guests as a function of the price, p .
- The *supply equation*, $S(p)$, gives the number of units I am willing to rent out as a function of the price, p . Suppose $S(p)$ is defined as follows:

$$S(p) = 50 - p$$

What is the equilibrium price (The price for which supply matches demand)? Consider only values which make sense with respect to the context of the application.

(20 pts) **Problem 6.** Consider the graph of $f(x)$ as shown below.



- Evaluate $\lim_{x \rightarrow 1^-} f(x)$.
- Evaluate $\lim_{x \rightarrow 1^+} f(x)$.
- Evaluate $\lim_{x \rightarrow 1} f(x)$.
- Is $f(x)$ continuous at $x = 1$ Why or why not?
- Evaluate $\lim_{x \rightarrow 2} f(x)$.
- Evaluate $f(2)$.
- Is $f(x)$ continuous at $x = 2$ Why or why not?
- Is $f(x)$ continuous on the interval $(-3, 0]$? Why or why not?