MA121 Elements of Calculus
Exam 1 Form 11

February 15, 2009

Instructions: Show all work relevant to the solution of each problem. i.e. no credit will be given for "just the answers." Please do *all* work in the Blue Books! There are **six** problems which carry a total 100 points. You will have until the end of class to complete this exam. Good luck!

(10 pts) **Problem 1.** Definitions and concepts.

a. If the limit as x approaches c from the left is the same as the limit as x approaches c from the right, what can I say about the general limit?

(*) It exists, and is equal to both "sided" limits.

b. True or false. If the function $f(x)$ is continuous on the interval $(-5, 3)$, then $f(x)$ is continuous at the point $x = 2$.

(*) True.

c. Draw the graph of a function which is not continous at the point $x = 3$.

(*) Place a hole or vertical asymptote at $x = 3$.

d. Draw the graph of a function for which the limit as x approaches 2 does not exist.

(*) Place a vertical asymptote or "sharp turn" at $x = 2$.

e. True or false. Consider the function $g(x)$. Suppose the average rate of change from A to B is positive. Then the slope of the secant line from A to B must be positive as well.

(*) True.

(20 pts) **Problem 2.** Consider the rational function $f(x) = \frac{x+9}{x^2-81}$.

a. What is the domain of $f(x)$?

(*) Solve $x^2 - 81 = 0$ to get $x = \pm 9$.
Domain is $\{x \mid x \neq \pm 9\}$.

b. Compute $\lim_{x \to 9} f(x)$.

(*) $\frac{x+9}{x^2-81} = \frac{1}{x-9}$
It is still not possible to plug in $x = 9$ and get a real number.
Hence, the limit does not exist.

c. Compute $\lim_{x\to\infty} f(x)$.

(*) The degree of the numerator is less than the degree of the denominator. So the limit is 0.

(20 pts) **Problem** 3. Consider the polynomial function $p(x) = x^2 - 8$.

a. Compute $p(x+h)$.

(*) $p(x+h) = (x+h)^2 - 8 = x^2 + 2hx + h^2 - 8$.

b. Find a simplified form of the difference quotient, $\frac{p(x+h)-p(x)}{h}$.

(*) $\frac{p(x+h)-p(x)}{h} = \frac{x^2+2hx+h^x-8-x^2+8}{h} = \frac{2hx+h^2}{h} = 2x + h$

c. Determine the average rate of change of $p(x)$ from $x = -2$ to $x = 2$.

(*) $\frac{p(2)-p(-2)}{2-(-2)} = \frac{-4+4}{4} = 0$.

d. Write the equation of the secant line from $x = -2$ to $x = 2$.

(*) The slope of the secant line is 0.
I have $y = 0x + b$.
Plug in $x = 2, y = p(2) = -4$ to get $-4 = b$.
$y = -4$.

(20 pts) **Problem** 4. Consider the piecewise function defined below.

$$f(x) = \begin{cases} x^3 - 1 & \text{if } x < 0; \\ x^2 + x & \text{if } x \geq 0. \end{cases}$$

a. Evaluate $f(0)$.

(*) $f(0) = 0$

b. Evaluate $\lim_{x\to 0^-} f(x)$.

(*) -1

c. Evaluate $\lim_{x\to 0} f(x)$.

(*) Does not exist (the right-hand limit is 0).

d. Is $f(x)$ continuous at $x = 0$?

(*) No. The limit does not exist.

e. Is $f(x)$ continuous on the interval $(-2, 2)$? Why or why not?

(*) No. Because it is not continuous at every point in the interval.

(10 pts) **Problem 5.** Suppose I am producing cats. When I charge 20 dollars for a bag, I sell 30 units. When I charge 40 dollars for a bag, I sell 15 units.

a. Write down the *slope* and *y-intercept* of the linear *demand equation*, $D(p)$, which gives the number of guests as a function of the price, p.

(*) Units sold y is a function of price p.

Slope is $\frac{y_2-y_1}{p_2-p_1} = \frac{40-20}{15-20} = -4$.

$D(p) = mp + b$. $m = -4$.

$D(p) = -4p + b$. When $p = 20$ then $D(p) = 30$.

$30 = -4(20) + b$

$b = 110$.

$D(p) = -4p + 110$.

b. The *supply equation*, $S(p)$, gives the number of units I am willing to rent out as a function of the price, p. Suppose $S(p)$ is defined as follows:

$$S(p) = 50 - p$$

What is the equillbrium price (The price for which supply matches demand)? Consider only values which make sense with respect to the context of the application.

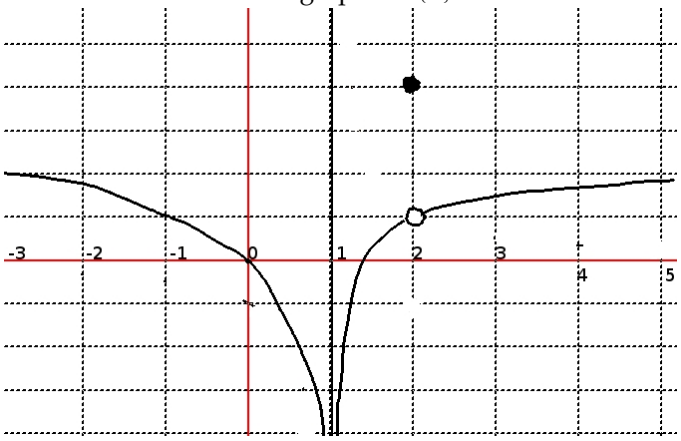
(*) $D(p) = S(p)$

$-4p + 110 = 50 - p$

$3p = 60$

$p = 20$.

(20 pts) **Problem 6.** Consider the graph of $f(x)$ as shown below.



a. Evaluate $\lim_{x \to 1^-} f(x)$.

(*) DNE or $-\infty$.

b. Evaluate $\lim_{x \to 1^+} f(x)$.

(*) DNE or $-\infty$.

c. Evaluate $\lim_{x \to 1} f(x)$.

(*) DNE or $-\infty$.

d. Is $f(x)$ continuous at $x = 1$ Why or why not?

(*) No. $f(1)$ is not a real number.

e. Evaluate $\lim_{x \to 2} f(x)$.

(*) 1

f. Evaluate $f(2)$.

(*) 4

g. Is $f(x)$ continuous at $x = 2$ Why or why not?

(*) No. The limit does not match $f(2)$.

h. Is $f(x)$ continuous on the interval $(-3, 0]$? Why or why not?

(*) Yes. $f(x)$ is continuous at every point in the interval.