

# MA121 Elements of Calculus

## Exam 1 Form 11

January 28, 2009

*Instructions:* Show all work relevant to the solution of each problem. i.e. no credit will be given for “just the answers.” Please do *all* work in the Blue Books! There are **six** problems which carry a total 100 points. You will have until the end of class to complete this exam. Good luck!

(10 pts) **Problem 1.** Definitions and concepts.

- If the limit as  $x$  approaches  $c$  from the left is the same as the limit as  $x$  approaches  $c$  from the right, what can I say about the general limit?
- True or false. If the function  $f(x)$  is continuous on the interval  $(-5, 3)$ , then  $f(x)$  is continuous at the point  $x = 2$ .
- Draw the graph of a function which is not continuous at the point  $x = 3$ .
- Draw the graph of a function for which the limit as  $x$  approaches 2 does not exist.
- True or false. Consider the function  $g(x)$ . Suppose the average rate of change from A to B is positive. Then the slope of the secant line from A to B must be positive as well.

(20 pts) **Problem 2.** Consider the rational function  $f(x) = \frac{x+9}{x^2-81}$ .

- What is the domain of  $f(x)$ ?
- Compute  $\lim_{x \rightarrow 9} f(x)$ .
- Compute  $\lim_{x \rightarrow \infty} f(x)$ .

(20 pts) **Problem 3.** Consider the polynomial function  $p(x) = x^2 - 8$ .

- Compute  $p(x+h)$ .
- Find a simplified form of the difference quotient,  $\frac{p(x+h)-p(x)}{h}$ .
- Determine the average rate of change of  $p(x)$  from  $x = -2$  to  $x = 2$ .
- Write the equation of the secant line from  $x = -2$  to  $x = 2$ .

(20 pts) **Problem 4.** Consider the piecewise function defined below.

$$f(x) = \begin{cases} x^3 - 1 & \text{if } x < 0; \\ x^2 + x & \text{if } x \geq 0. \end{cases}$$

- Evaluate  $f(0)$ .
- Evaluate  $\lim_{x \rightarrow 0^-} f(x)$ .
- Evaluate  $\lim_{x \rightarrow 0} f(x)$ .
- Is  $f(x)$  continuous at  $x = 0$ ?
- Is  $f(x)$  continuous on the interval  $(-2, 2)$ ? Why or why not?

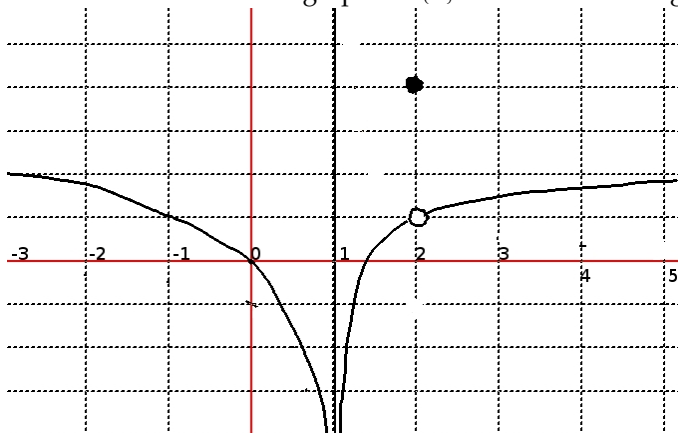
(10 pts) **Problem 5.** Suppose I am producing cats. When I charge 20 dollars for a bag, I sell 30 units. When I charge 40 dollars for a bag, I sell 15 units.

- Write down the *slope* and *y-intercept* of the linear *demand equation*,  $D(p)$ , which gives the number of guests as a function of the price,  $p$ .
- The *supply equation*,  $S(p)$ , gives the number of units I am willing to rent out as a function of the price,  $p$ . Suppose  $S(p)$  is defined as follows:

$$S(p) = 50 - p$$

What is the equilibrium price (The price for which supply matches demand)? Consider only values which make sense with respect to the context of the application.

(20 pts) **Problem 6.** Consider the graph of  $f(x)$  as shown to the right.



- Evaluate  $\lim_{x \rightarrow 1^-} f(x)$ .
- Evaluate  $\lim_{x \rightarrow 1^+} f(x)$ .
- Evaluate  $\lim_{x \rightarrow 1} f(x)$ .
- Is  $f(x)$  continuous at  $x = 1$  Why or why not?
- Evaluate  $\lim_{x \rightarrow 2} f(x)$ .
- Evaluate  $f(2)$ .
- Is  $f(x)$  continuous at  $x = 2$  Why or why not?
- Is  $f(x)$  continuous on the interval  $(-3, 0]$ ? Why or why not?