MA131 Calculus for Life and Management Sciences A
Exam 3 Review Questions

November 17, 2009

1. Definitions and Concepts.

   a. Give a function, f, which satisfies the relationship $\frac{df}{dx} = 1$.

   b. Why does it not matter if I forget the "+C" when using *definite* integration?

   c. Find the error in my proof that $0 = 1$.

   Let $f(x) = 2x$ and $g(x) = 2x+1$. Then $f'(x) = 2$ and $g'(x) = 2$. Anti-differentiating, $f(x) = 2x + C$ and $g(x) = 2x + C$. Then $f(x) = g(x)$. So it follows $2x = 2x + 1$. Subtrating $2x$ from both sides of the equation, I find $0 = 1$.

   e. Give an example of a function, $f(x)$ such that the area bounded by the x-axis, $f(x)$, $x = 0$, and $x = 1$ can be determined exactly using one rectangle (as opposed to an infinite series of rectangles).

   f. Give an example of a function, $f(x)$, such that $\int f(x) = 3x^2 + 2x$.

2. Find $\int f(x)dx$ for each of the following.

   a. $f(x) = 3x^2 + 5x + 1$

   b. $f(x) = \frac{44}{x}$

   c. $f(x) = \frac{1}{x}(\ln(3x))^5$

   d. $f(x) = \frac{9x^2+3}{3x^3+3x}$

   e. $f(x) = \sqrt{x}$

3. For each of the following, use Riemann Sums (with "left-hand rule") with 4 rectangles to approximate the area under $f(x)$, bounded on the left by 0 and on the right by 2. Then find $\int_0^2 f(x)dx$ for each of the following. Finally, give the average value of the function over the interval from 0 to 2.

   a. $f(x) = x^2 + x$

   b. $f(x) = xe^{x^2}$

4. Find the area of the region bounded by the x-axis and the curve $f(x) = -(x^2) + 4$.

5. Find the volume of the solid constructed by rotating the above region about the x-axis.

6. Find the area of the region bounded by the curves $f(x) = \sqrt{x}$ and $f(x) = x^2$.

7. Consider the following supply function, $S(x)$, and demand function, $D(x)$, where price is a function of quantity.
$$S(x) = x^2 - 2x + 2,\ D(x) = -2x + 11$$
   a. Find the equillibrium point (the value of x such that supply equals demand).
   b. Find the consumer's surplus at this point.
   c. Find the producer's surplus at this point.

8. Find the area under the piece-wise defined function from $x = 0$ to $x = 99$.
$$f(x) = \begin{cases} 1 & \text{if } x \leq 50; \\ 2 & \text{if } x > 50. \end{cases}$$

9. Find the amount of continuous money flow in which 5000 per year is being invested at 6%, compounded continuously, for 45 years.

10. In 1897 the world's consumption of Watsonium-199 was $55,000\ ft^3$. The amount used has steadily increased at an average rate of 10% per year. What is the consumption rate in 2007? What was the total amount of Watsonium-199 consumed?

11. Find $\lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x_i$ where $f(x) = 2x$ on the closed interval $[3, 5]$.

12. (40) Compute each of the following:
   a. $\int 6x^3 + 6\sqrt{x}dx$
   b. $\int \frac{5}{x}dx$
   c. $\int e^x + 2e^x dx$
   d. $\int \frac{6x^2+5}{2x^3+5x+1}dx$
   e. $\int 6xe^{x^2+1}dx$
   f. $\int 15x^2(x^3+2)^9 dx$
   g. $\int_0^1 x^2 + x dx$
   h. $\int_1^2 \frac{x-1}{x^2-x}dx$

13. (20) Watsonovia Bank offers an account which pays interest at the rate of 6%, compounded continuously.
   a. I deposit 10,000 dollars into an account. What is its value after 20 years?
   b. Find the future value of a continous money flow of 1,000 dollars per year for 40 years.