MA131 Calculus for Life and Management Sciences A
Exam 4 Form 41

November 2009

Instructions: Show all work relevant to the solution of each problem. i.e. no credit will be given for "just the answers." Please do *all* work in the Blue Books! There are **seven** problems which carry a total 100 points. You will have until the end of class to complete this exam. Good luck!

(15 pts) **Problem 1.** Let $f(x) = \frac{1}{4}x^3 + \sqrt{x}$.

a. Approximate the area under the curve $f(x)$, bounded on the left by $x = 0$, and on the right by $x = 1$, using Riemann Sums. Use the left-handed rule and 2 rectangles.

(*) Rectangle 1. Area is $.5f(0) = 0$.
Rectangle 2. Area is $.5f(.5) = .369178$.

Total area is approximately .369178.

b. Find the exact area of the region described.

(*) $F(x) = \frac{2}{3}x^{\frac{3}{2}} + \frac{1}{16}x^4$.
$F(1) - F(0) = \frac{35}{48}$.

(15 pts) **Problem 2.** Let $f(x) = \frac{1}{x}$

a. Compute $F(x)$, the anti-derivative of $f(x)$.

(*) $F(x) = \ln|x|$.

b. Let a region be bounded by the x-axis, $f(x)$, $x = 1$, and $x = 2$. Compute the volume of the solid obtained by rotating this region about the x-axis.

(*) $\int_1^2 \pi(\frac{1}{x})^2 dx = .5\pi$

(10 pts) **Problem 3.** Compute the area bounded by the curve $f(x) = 1 - x$, the x-axis, $x = -1$ and $x = 1$.

(*) $\int_{-1}^1 1 - x dx = 2$

(15 pts) **Problem 4.** Let $f(x) = 2x\sqrt{x^2 + 5}$.

a. Compute $\int_1^3 f(x)dx$.

(*) Use substitution. Let $u = x^2 + 5$.

25.1242

b. Find the average value of $f(x)$ over the interval $[1, 3]$.

(*) Average value is $\frac{1}{b-a}\int_1^3 f(x)dx = \frac{1}{2} * 24.1242 = 12.5621$

(15 pts) **Problem** 5. Find the area of the region bounded by the curves $f(x) = x^4$ and $g(x) = x^5$.

(*) $f(x)$ and $g(x)$ intersect at $x = 0$ and $x = 1$. The function $f(x)$ is higher in this domain. Then

$\int_0^1 f(x) - g(x)dx = \frac{1}{30}$

(15 pts) **Problem** 6. Watsonium-Q is a fuel used for rockets in the late 21st century. Usage is given in tons according to the formula $A(t) = 50000e^{.05t}$, where t is years since 2070.

a. Calculate the total usage of Watsonium-Q over the years 2070 to 2100.

(*) The total usage is given by $\int_0^{30} A(t)dt$, which using the trick for integrating an exponential model (or using u-substitution), we get 3481690 tons.

b. Suppose worldwide reserves of Watsonium-Q is a fixed 4.1 million tons. Assuming no future growth in reserves, is 4.1 million tons enough to meet the predicted demand over the years 2070 - 2100?

(*) Yes.

(15 pts) **Problem** 7. The birth rate $b(t)$ and death rate $d(t)$ for Watson City is given as the following functions of t (years since 2000).

$$b(t) = 713 - 9t^2, \text{ and } d(t) = 337 + 32t^2$$

a. Find the total number of births from the years 2000 to 2010.

(*) Total births is $\int_0^{10} b(t)dt = 4130$.

b. Find the total number of deaths from the years 2000 to 2010.

(*) Total deaths is $\int_0^{10} d(t)dt = 14038$.

c. The population in the year 2000 was 50,000. Estimate the population in the year 2010.

(*) The new population will be CURRENT POPULATION + TOTAL BIRTHS - TOTAL DEATHS, which is 40092.