MA131 Calculus for Life and Management Sciences A
Exam 3 Form 31

November 2009

Instructions: There are **seven** problems, each of which carry a total of 16 points. Work **six** of the problems **and throw out one**. You may work all seven, but you must clearly indicate which is to not be graded. There is no extra credit for working all seven, nor will the best six of seven scores be used. If you work all seven and do not indicate which is to be skipped, then the last problem will be omitted by default. I will add 4 points to your score for "free," for a total of 100 points. Show all work relevant to the solution of each problem. i.e. no credit will be given for "just the answers." Please do *all* work in the Blue Books! You will have until the end of class to complete this exam. Good luck!

(16 pts) **Problem 1.** Compute the derivative in each case

a. $\frac{d}{dx}\ln(\frac{x^2-x-6}{x-3})$ **b.** $\frac{d}{dx}\sqrt{x}\,e^{x^2}$

(*) a. Note that $x^2 - x - 6 = (x-3)(x+2)$. Then

$\frac{d}{dx}\ln(\frac{x^2-x-6}{x-3}) = \frac{d}{dx}\ln(x+2)$

The derivative of $x+2$ is 1. So $\frac{d}{dx}\ln(x+2) = \frac{1}{x+2}$

b. I'll have to use the product rule here. First write $\sqrt{x}$ as $x^{.5}$. Then I have

$\frac{d}{dx}x^{.5} \times e^{x^2}$

The first function is $x^{.5}$, and its derivative is $.5x^{-.5}$. The second function is e^{x^2} and its derivative is $2xe^{x^2}$.

So by the product rule, $\frac{d}{dx}x^{.5} \times e^{x^2} = .5x^{-.5}e^{x^2} + x^{.5}2xe^{x^2}$

(16 pts) **Problem 2.** Let $f(3) = 4, f'(3) = 7, g(3) = 3, g'(3) = 2$. Find $h(3)$ and $h'(3)$ in each of the cases

a. $h(x) = \frac{x}{f(x)+g(x)}$ **b.** $h(x) = e^{g(x)}$

(*) a. I need to use the quotient rule. The numerator is x, and its derivative is 1. The denominator is $f(x) + g(x)$ and its derivative is $f'(x) + g'(x)$.

So I have $h'(x) = \frac{(f(x)+g(x))(1)-(x)(f'(x)+g'(x))}{(f(x)+g(x))^2}$.

Then $h'(3) = \frac{(f(3)+g(3))(1)-(3)(f'(3)+g'(3))}{(f(3)+g(3))^2}$.

b. I recall that the shortcut to the chain rule for exponential functions gives me $h'(x) = g'(x)e^{g(x)}$. Then $h'(3) = g'(3)e^{g(x)}$.

(16 pts) **Problem 3.** In October 2009 Milton's consumption of cat litter is 40lb per month, and is increasing at the rate of 2lb per month. The cost per pound is .75 dollars, and is increasing at the rate of .1 dollars per month. What is Milton's total cat litter bill for the month? At what rate is it increasing?

(*) Let $f(t)$ represent Milton's consumption of litter in pounds, and $g(t)$ represent the price of litter per pound. Then the total bill is $T(t) = f(t)g(t)$.

I want to find the *rate of change* of the litter bill, or $T'(t)$.

To do so, I need to use the product rule. $T'(t) = \frac{d}{dt}f(t)g(t) = f'(t)g(t) + f(t)g'(t)$

$f'(t)$ is the rate of change of consumption, which is 2. $g'(t)$ is the rate of change of price, which is .1.

Then the rate of change of the bill is $f'(t)g(t) + f(t)g'(t) = (2)(.75) + (40)(.1)$

(16 pts) **Problem 4.** The bacteria known to cause deadly Watsonitis are being studied. A scientist places 5,000 bacteria in a dish. Six hours later, the number of bacteria has increased to 11,300. It is known that the population of bacteria grow according to the model $\frac{dP}{dt} = kP(t)$, where $P(t)$ is the population after t hours. Construct the function $P(t)$ which gives the population after t hours and satisfies the differential equation given. Use your function to estimate the population of the colony after one day (24 hours).

(*) $P(t) = 5000e^{kt}$.

To find k I know that when $t = 6$ then $P(t) = 11300$. So I have the equation $11300 = 5000e^{6k}$. I can solve for k by taking the log of both sides of the equation, to get $k = .135894$.

Then $A(t) = 5000e^{.135894t}$.

The population after one day is $A(24) = 130438$.

(16 pts) **Problem 5.** Archaeologists believe they have found the lost continent of Atlantis. To help test their conjecture, they obtain samples from bones found at the site. They determine the bones to have 70% of their original amount of Carbon-14. Assuming a half-life of 5,700 years and that Carbon-14 decays exponentially, determine the age of the bones.

(*) The decay constant (k) can be found quickly by using the formula $k = \frac{\ln(2)}{T}$, where T is the half life. This means $k = \frac{\ln(2)}{5700} = .000121605$.

So I want to know t when $.7C = Ce^{-.000121605t}$. The C's cancel and I solve for t by taking the log of both sides of the equation. I get $t = 2933.07$.

(16 pts) **Problem** 6. A developer is considering buying a rectangular section of land alongside a lake for the site of a condominium. The city determines the property tax in the following way. Each foot of land bordering the lake incurs a tax of 40 dollars per month. Each foot of land alongside the other edges incurs a tax of 15 dollars per month. The developer wants to determine the largest area of land she can buy so that the monthly property tax does not exceed 5000 dollars. Determine the optimal dimensions of the site.

(*) Area is given by $A = lw$. Now I can't differentiate this function as is, so I need to find and use a constraint equation to write one of the variables in terms of the other. Let one of the two length-wise dimensions be bordering the lake. Then the tax is $55l + 30w$. (40 for the lakeside length, 15 for the other length, and 15 for each of the two width-wise lengths). I require $55l+30w = 5000$. Solve for w to get $w = 166.67-1.833l$.

My area is now a function of l after substituting: $A(l) = l(166.67 - 1.833l)$.

Then $A'(l) = 166.67 - 3.666l$.
Solve $A'(l) = 0$ to get $l = 45.4637$.

If $l = 45.4637$ then $w = 83.335$.

(16 pts) **Problem** 7. Consider the function $f(x) = 2x^3 + 3x^2 - 12x + 5$. Find the absolute minimum and maximum of $f(x)$ over the domain $[-3, 3]$.

(*) $f'(x) = 6x^2 + 6x - 12$.

Solve $f'(x) = 0$ to get $x = -2$ and $x = 1$.

I have the y values $f(-3) = 14$, $f(-2) = 25$, $f(1) = -2$, and $f(3) = 50$. The max occurs at $(3, 50)$. The min occurs at $(1, -2)$.