

# MA131 Calculus for Life and Management Sciences A

## Exam 1 Review Questions

September 30, 2009

1. Find the first and second derivatives for each of the following functions.

a.  $f(x) = x^2 + 2x + 1$

b.  $f(x) = \sqrt{x}$

c.  $f(x) = \frac{1}{\sqrt{x}}$

d.  $f(x) = (x^2 + 2x + 5)^8$

e.  $f(x) = (2x + 7)^{-3}$

f.  $f(x) = \frac{1}{\sqrt{2x+1}}$

2. Let  $f(3) = 4$  and  $f'(3) = 6$ . Let  $g(3) = 7$  and  $g'(3) = 2$ . Determine  $h(3)$  where  $h(x)$  is defined as follows:

a.  $h(x) = f(x) + g(x)$ .

b.  $h(x) = (f(x))^3$ .

c.  $h(x) = \frac{1}{g(x)}$ .

d.  $h(x) = (f(x) + g(x))^3$ .

3. Evaluate the limits that exist. If the limit does not exist, write "DNE". You do not need to distinguish between "Infinity" and "DNE".

a.  $\lim_{x \rightarrow 1} \frac{1}{x-1}$

b.  $\lim_{x \rightarrow \infty} \frac{1}{x-1}$

c.  $\lim_{x \rightarrow 4} \frac{x^2-16}{x-4}$

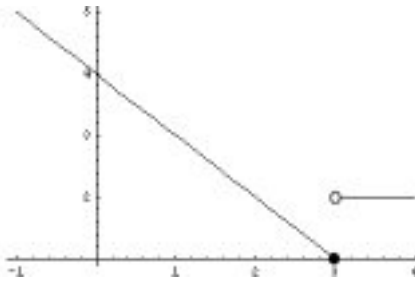
d.  $\lim_{h \rightarrow 0} hx^2 + h^2x + x + 3$

e.  $\lim_{x \rightarrow \infty} \frac{5x^2-19x+2}{x^2-3}$

f.  $\lim_{x \rightarrow \infty} \frac{x^3+9x^2+5}{765942x^2+34245343x+10000000000}$

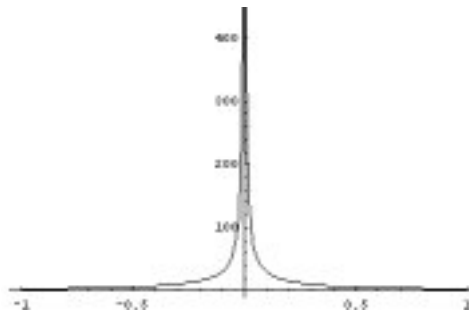
g.  $\lim_{x \rightarrow 0} |x|$ .

h.  $\lim_{x \rightarrow 1} \sqrt{x^2 + x - 6}$



4. The graph of  $f(x)$  is given to the right.

- Evaluate  $\lim_{x \rightarrow 3^-} f(x)$ .
- Evaluate  $\lim_{x \rightarrow 3^+} f(x)$ .
- Evaluate  $\lim_{x \rightarrow 3} f(x)$ .
- Is  $f(x)$  continuous at  $x = 3$ ?
- Is  $f(x)$  continuous on the interval  $[-1, 2]$ ?



The graph of  $g(x)$  is given to the right.

- Evaluate  $\lim_{x \rightarrow 0^-} g(x)$ .
  - Evaluate  $\lim_{x \rightarrow 0^+} g(x)$ .
  - Is  $g(x)$  continuous on the interval  $[-1, 1]$ ?
5. Watson Enterprises is considering producing and selling cat-proof keyboards. They plan to sell each keyboard for 40 dollars. Materials for each keyboard cost 20 dollars, and the machines to produce the keyboards cost 100,000 dollars. The machines must only be purchased once.
- Set up the linear function  $C(x)$  which gives the total cost of producing  $x$  keyboards.
- The cost per unit is the cost of the materials plus an equal fraction of the cost of the machine. So, for instance, if five keyboards are produced, the cost per keyboard is 20 dollars plus one-fifth of the cost of the machine (20,000 dollars). So the cost per unit if  $x$  units are produced is given by
- $$M(x) = \frac{20x + 100000}{x}$$
- What is the cost per unit if 10 keyboards are produced?
  - Compute  $\lim_{x \rightarrow \infty} M(x)$ .

- d. What does your answer to (c) tell us about the cost per unit?
- e. Give a model for the marginal cost as a function of  $x$ .
6. Suppose I am producing cats. When I charge 20 dollars for a bag, I sell 30 units. When I charge 40 dollars for a bag, I sell 15 units.
- Write down the *slope* and *y-intercept* of the linear *demand equation*,  $D(p)$ , which gives the number of sales as a function of the price,  $p$ .
  - Write down a model giving the *revenue* (product of sales and price) as a function of price.
  - Write down a model for the marginal revenue.
  - Suppose I charge 19 dollars per bag. Use your model from (c) to determine if I would increase or decrease revenue by raising the price.
7. Consider the piecewise function defined below.

$$f(x) = \begin{cases} x + 2 & \text{if } x \neq 4; \\ 8 & \text{if } x = 4. \end{cases}$$

- Evaluate  $f(4)$ .
  - Evaluate  $\lim_{x \rightarrow 4} f(x)$ .
  - Is  $f(x)$  continuous at  $x = 3$ ? What about  $x = 4$ ?
  - Is  $f(x)$  continuous on the interval  $[0, 10]$ ? Why or why not?
8. Let  $f(x) = x^3 - 12x + 16$ .
- Locate the relative extrema on  $f(x)$ .
  - On what interval(s) is  $f(x)$  increasing?
  - Locate the inflection points of  $f(x)$ .
  - On what interval(s) is  $f(x)$  concave up?
9. The graph of  $g(x)$  is given below.
- At which points is  $g(x)$  *not* continuous?
  - At which points is  $g(x)$  *not* differentiable?
  - At which points is the slope of the line tangent to  $g(x)$  horizontal?
  - Which points, if any, are the critical points?
  - Which points, if any, are relative extrema?
  - Which points, if any, are relative maxima?
  - Which point, if any, is the relative minima?

