

MA131 Calculus for Life and Management Sciences A

Exam 2 Form 22 Solution

October 2009

Instructions: Show all work relevant to the solution of each problem. i.e. no credit will be given for “just the answers.” Please do *all* work in the Blue Books! There are **six** problems which carry a total 100 points. You will have until the end of class to complete this exam. Good luck!

(10 pts) **Problem 1.** Find the first and second derivative of the function $f(x) = \sqrt{5x+4}$. Write $f(x) = (5x+4)^{.5}$

Then use the general power rule to write $f'(x) = .5(5x+4)^{-.5}(5) = 2.5(5x+4)^{-.5}$.

To find the second derivative, repeat the general power rule.

$$f'(x) = -1.25(5x+4)^{-1.5}(5)$$

(15 pts) **Problem 2.** Evaluate each of the following limits. If the limit does not evaluate to a finite real number, write “DNE.”

a. $\lim_{x \rightarrow 4} \frac{x^2 - 2x - 8}{x - 4}$

(*) Notice you can write the rational as $\frac{(x-4)(x+2)}{x-4} = x + 2$.

Let $x = 4$ to find that the limit is 6.

b. $\lim_{x \rightarrow 0} \sqrt{4x^2 + 2x + 1}$

(*) The expression cannot be simplified further. Let $x = 0$ to find the limit is $\sqrt{1} = 1$.

c. $\lim_{x \rightarrow \infty} \frac{7x^2 - 121x + 266}{4x^2 + 9x - 3}$

(*) The degrees of the numerator and denominator match. So the limit is $\frac{7}{4}$.

(10 pts) **Problem 3.** Let $f(7) = 2$, $f'(7) = 3$, $g(7) = 6$, and $g'(7) = 5$.

a. Find $h(7)$ and $h'(7)$ where $h(x) = f(x) + 3g(x)$.

$$(*) h(7) = f(7) + 3g(7) = 2 + 3(6) = 20$$

$$h'(x) = f'(x) + 3g'(x).$$

$$h'(7) = 3 + 3(5) = 18.$$

- b. Find $h(7)$ and $h'(7)$ where $h(x) = \frac{1}{g(x)}$.

$$(*) h(7) = \frac{1}{g(7)} = \frac{1}{5}.$$

To find $h'(x)$, first write $h(x) = (g(x))^{-.5}$. Then $h'(x) = .5(g(x))^{-.5}g'(x)$.
 $h'(7) = .5(5)^{-.5}6$

(25 pts) **Problem 4.** Let $f(x) = \frac{1}{3}x^3 - 4x$

- a. Locate the relative extrema on $f(x)$.

(*) Write $f'(x) = x^2 - 4$.
Solve $f'(x) = 0$ to get $(x + 4)(x - 4) = 0$.
Then $x = 4$ and $x = -4$.

- b. Classify the relative extrema as relative maxima, minima, or neither.

(*) Note that I can get this answer from the solution to part c. I can also use the second derivative test.

$f''(x) = 2x$.
 $f''(-4) = -8 < 0$, so I have a max at $x = -4$.
 $f''(4) = 8 > 0$, so I have a min at $x = 4$.

- c. Identify the interval(s) for which $f(x)$ is increasing.

(*) If I partition the real number line using the relative extrema, I have three intervals. For each interval I will pick a value of x inside that interval, plug it into the *first derivative*, and make note of the sign. If it is positive, $f(x)$ is increasing on that interval. If it is negative, then $f(x)$ is decreasing.

$(-\infty, -4)$. Let $x = -10$. Then $f'(x) > 0$. Increasing.
 $(-4, 4)$. Let $x = 0$. Then $f'(x) < 0$. Decreasing.
 $(4, \infty)$. Let $x = 10$. Then $f'(x) > 0$. Increasing.

- d. Locate the inflection point(s) on $f(x)$.

(*) Solve $f''(x) = 0$ to get $x = 0$. So I have an inflection point at $x = 0$.

- e. Identify the interval(s) for which $f(x)$ is concave down.

(*) I will partition the real number line using the inflection points. For each interval I pick a number in the interval, plug it into the *second derivative*, and note the sign. If it is positive, $f(x)$ is concave up. If it is negative, $f(x)$ is concave down.

$(-\infty, 0)$. Let $x = -50$. Then $f''(x) < 0$. Concave down.
 $(0, \infty)$. Let $x = 50$. Then $f''(x) > 0$. Concave up.

(10 pts) **Problem 5.** The revenue earned by producing and selling x units of the board game *Watsonopoly* can be modelled by the revenue function

$$R(x) = 2000 - .1x - 1000(.2x + 7)^{-1}$$

- a. Compute a function $R'(x)$ which gives the *marginal revenue* as a function of x .

(*) Use the general power rule.
 $R'(x) = -.1 + 1000(.2x + 7)^{-2}$.

- b. Suppose I am producing 400 units. Should I expect revenues to increase or decrease by increasing production? Justify your answer.

(*) $R'(400) < 0$, so at $x = 400$, revenues decrease as x increases. Then I should expect revenues to decrease.

(30 pts) **Problem 6.** Consider the graph of $f(x)$ below.

a. At which labelled points does $\lim_{x \rightarrow \text{(labelled point)}} f(x)$ exist?

(*) A, B, C, D, E, G, H

b. At which point(s) is $f(x)$ continuous?

(*) A, B, C, D, E, G, H

c. Is $f(x)$ continuous on the interval $(2, 4)$? Why or why not?

(*) No. Because $f(x)$ is not continuous at a point in this interval.

d. At which point(s) is $f(x)$ differentiable?

(*) A, B, C, D, E, H

e. At label A is $f'(x)$ positive, negative, or zero?

(*) Negative

f. At which point(s) is there a relative extrema?

(*) B, D

g. At which point(s) is there a relative maxima?

(*) D

h. At label B is $f''(x)$ positive, negative, or zero?

(*) Positive

i. At label C is $f''(x)$ positive, negative, or zero?

(*) 0

j. State the value of $f'(x)$ at label D.

(*) 0

