MA131 Calculus for Life and Management Sciences A
Exam 2 Form 21 Solution

October 2009

Instructions: Show all work relevant to the solution of each problem. i.e. no credit will be given for "just the answers." Please do *all* work in the Blue Books! There are **six** problems which carry a total 100 points. You will have until the end of class to complete this exam. Good luck!

(10 pts) **Problem 1.** Find the first and second derivative of the function $f(x) = \sqrt{4x+5}$. Write $f(x) = (4x+5)^{.5}$

Then use the general power rule to write $f'(x) = .5(4x+5)^{-.5}(4) = 2(4x+5)^{-.5}$.

To find the second derivative, repeat the general power rule.

$f'(x) = -1(4x+5)^{-1.5}(4)$

(15 pts) **Problem 2.** Evaluate each of the following limits. If the limit does not evaluate to a finite real number, write "DNE."

a. $\lim_{x\to 4} \frac{x^2-2x-8}{x-4}$

(*) Notice you can write the rational as $\frac{(x-4)(x+2)}{x-4} = x+2$.

Let $x = 4$ to find that the limit is 6.

b. $\lim_{x\to 0} \sqrt{4x^2+2x+1}$

(*) The expression cannot be simplified further. Let $x = 0$ to find the limit is $\sqrt{1} = 1$.

c. $\lim_{x\to\infty} \frac{7x^2-121x+266}{4x^2+9x-3}$

(*) The degrees of the numerator and denominator match. So the limit is $\frac{7}{4}$.

(10 pts) **Problem 3.** Let $f(7) = 3$, $f'(7) = 2$, $g(7) = 5$, and $g'(7) = 6$.

a. Find $h(7)$ and $h'(7)$ where $h(x) = f(x) + 3g(x)$.

(*) $h(7) = f(7) + 3g(7) = 3 + 3(5) = 18$

$h'(x) = f'(x) + 3g'(x)$.
$h'(7) = 2 + 3(6) = 20$.

b. Find $h(7)$ and $h'(7)$ where $h(x) = \frac{1}{g(x)}$.

> **(*)** $h(7) = \frac{1}{g(7)} = \frac{1}{5}$.
>
> To find $h'(x)$, first write $h(x) = (g(x))^{.5}$. Then $h'(x) = .5(g(x))^{-.5}g'(x)$.
> $h'(7) = .5(5)^{-.5}6$

(25 pts) **Problem 4.** Let $f(x) = \frac{1}{3}x^3 - 4x$

a. Locate the relative extrema on $f(x)$.

> **(*)** Write $f'(x) = x^2 - 4$.
> Solve $f'(x) = 0$ to get $(x+4)(x-4) = 0$.
> Then $x = 4$ and $x = -4$.

b. Classify the relative extrema as relative maxima, minima, or neither.

> **(*)** Note that I can get this answer from the solution to part c. I can also use the second derivative test.
>
> $f''(x) = 2x$.
> $f''(-4) = -8 < 0$, so I have a max at $x = -8$.
> $f''(4) = 8 > 0$, so I have a min at $x = 8$.

c. Identify the interval(s) for which $f(x)$ is increasing.

> **(*)** If I partition the real number line using the relative extrema, I have three intervals. For each interval I will pick a value of x inside that interval, plug it into the *first derivative*, and make note of the sign. If it is positive, $f(x)$ is increasing on that interval. If it is negative, then $f(x)$ is decreasing.
>
> $(-\infty, -4)$. Let $x = -10$. Then $f'(x) > 0$. Increasing.
> $(-4, 4)$. Let $x = 0$. Then $f'(x) < 0$. Decreasing.
> $(4, \infty)$. Let $x = 10$. Then $f'(x) > 0$. Increasing.

d. Locate the inflection point(s) on $f(x)$.

> **(*)** Solve $f''(x) = 0$ to get $x = 0$. So I have an inflection point at $x = 0$.

e. Identify the interval(s) for which $f(x)$ is concave down.

> **(*)** I will partition the real number line using the inflection points. For each interval I pick a number in the interval, plug it into the *second derivative*, and note the sign. If it is positive, $f(x)$ is concave up. If it is negative, $f(x)$ is concave down.
>
> $(-\infty, 0)$. Let $x = -50$. Then $f''(x) < 0$. Concave down.
> $(0, \infty)$. Let $x = 50$. Then $f''(x) > 0$. Concave up.

(10 pts) **Problem 5.** The revenue earned by producing and selling x units of the board game *Watsonopoly* can be modelled by the revenue function

$$R(x) = 2000 - .1x - 1000(.2x + 7)^{-1}$$

a. Compute a function $R'(x)$ which gives the *marginal revenue* as a function of x.

> **(*)** Use the general power rule.
> $R'(x) = -.1 + 1000(.2x + 7)^{-2}$.

b. Suppose I am producing 400 units. Should I expect revenues to increase or decrease by increasing production? Justify your answer.

> **(*)** $R'(400) < 0$, so at $x = 400$, revenues decrease as x increases. Then I should expect revenues to decrease.

(30 pts) **Problem** 6. Consider the graph of $f(x)$ below.

a. At which labelled points does $\lim_{x \to \langle \text{labelled point} \rangle} f(x)$ exist?

(*) A, B, C, D, E, G, H

b. At which point(s) is $f(x)$ continuous?

(*) A, B, C, D, E, G, H

c. Is $f(x)$ continuous on the interval $(2, 4)$? Why or why not?

(*) No. Because $f(x)$ is not continuous at a point in this interval.

d. At which point(s) is $f(x)$ differentiable?

(*) A, B, C, D, E, H

e. At label A is $f'(x)$ positive, negative, or zero?

(*) Negative

f. At which point(s) is there a relative extrema?

(*) B, D

g. At which point(s) is there a relative maxima?

(*) D

h. At label B is $f''(x)$ positive, negative, or zero?

(*) Positive

i. At label C is $f''(x)$ positive, negative, or zero?

(*) 0

j. State the value of $f'(x)$ at label D.

(*) 0