Section 8.3: The Integral and Comparison Tests; Estimating Sums

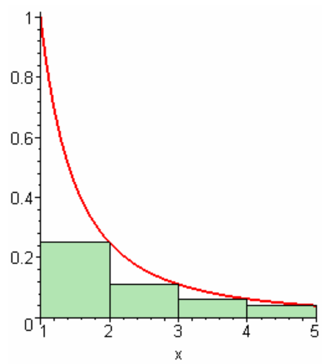
Discussion: In section 8.2, we were able to determine the exact sum of geometric series since we could find a simple formula for the n^{th} partial sum s_n. In general, it is not easy to compute $\lim_{n \to \infty} s_n$; however, we can often use tests to determine if a series converges or diverges, and in some cases, we can find a good estimate of the sum.

Note: The tests in this section will apply to ____.
This means that ____
____.

- **The Integral Test**:

Why does the test work?

From the figure below, notice that the area under the curve $y = f(x)$ as x goes form 1 to ∞ is greater thatn the sum of the areas of the rectangles.



Notice that the first rectangle has height $f(2)$, and $f(2) = a_2$. The second rectangle has height $f(3) = a_3$, etc. Also, each rectangle has width = 1, so the area of rectangle 1 is a_2, rectangle 2 is a_3, etc. Thus, the n^{th} rectangle has area a_{n+1}.

Hence, ____.

If $\int_1^{\infty} f(x)dx$ is finite,.

- **Example:** Recall, from section 8.2, we said that the harmonic series given by $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges. Use the integral test to show this.

- The harmonic series is part of a family of series called *p*-series. A ____.
  - o
  - o
  - o
- ***p*-series test:**

  Proof:

- **Example:** Use the Integral Test to determine if the $\sum_{n=1}^{\infty} 7n^2 e^{-n^3}$ converges.

Discussion: If we are able to use the Integral Test to show that a series $\sum_{n=1}^{\infty} a_n$ converges, we can find an approximation to the sum *s* of the series. Any partial sum s_n will approximate the true value *s* since $\lim_{n \to \infty} s_n = s$, but how good will the approximation be?

- **Remainder Estimate for the Integral Test**: If $\sum_{n=1}^{\infty} a_n$ converges by the Integral Test, and $R_n = s - s_n$ ______________

____.

- **Example:** Use the integral test to show that $\sum_{n=1}^{\infty} e^{-n}$ converges. Then, using Maple, approximate the sum of the series by using the partial sum s_{20}. Estimate the error involved in this approximation.

- Find an integer n such that, using s_n as an approximation of the series $\sum_{n=1}^{\infty} \frac{7}{n^4}$, the maximum possible error is at most 0.00001.

- **Individual Comparison Tests:** ____.
  - o
  - o

  Note:

- **Example**: Determine which of the following series converge or diverge.
  - o $\sum_{n=1}^{\infty} \frac{1}{2+3^n}$

- $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}-1}$

- $\sum_{n=1}^{\infty} \frac{1}{n+1}$

- **Limit Comparison Test**: ____
____.

- **Example**: Use the limit comparison test to determine the convergence of the series $\sum_{n=2}^{\infty} \frac{1}{n^3 - n}$.

- **Example**: Use the limit comparison test to determine the convergence of the series $\sum_{n=1}^{\infty} \frac{n^3+1}{n^5+4n^3+3}$.

Section 8.4: Other Convergence Tests

Discussion: In Section 8.3, we considered the convergence of series with strictly positive terms. We will now look at series whose terms are not necessarily positive.

- An **alternating series** is ____
____.
- **Example**:
- **The Alternating Series Test:** ____

(a)

(b)

then

- **Example**: Does $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n}$ converge?
- **Example**: Use the alternating series test to determine the convergence of the series $\sum_{n=1}^{\infty}\frac{(-1)^n 2^n}{n^2}$.