Chapter 8: Infinite Sequences and Series

Note: The Visual Calculus website has material on Sequences and Series. Here's the address: http://archives.math.utk.edu/visual.calculus/6/index.html.

Why are sequences and series important to calculus? We can represent functions as sums of infinite series. Newton often found areas by first expressing a function as an infinite series and then integrating each term of the series. This also allows us to integrate functions that we previously couldn't such as e^{-x^2} . Many functions in physics and chemistry are defined as sums of series. Thus, it is beneficial to be exposed to the basic concepts of convergence of infinite sequences and series.

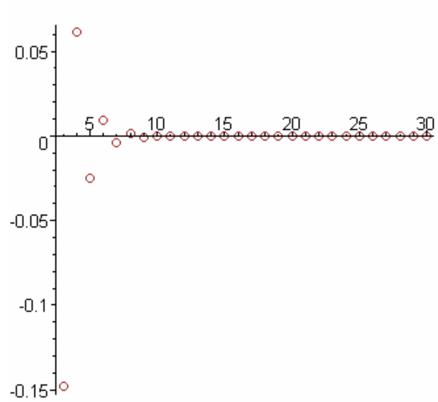
Section 8.1: Sequences

- **Sequence ➔**
  - o
  - o
  - o
- **Infinite Sequence ➔**
- **Notation:**
- **Note:**
- Some sequences are defined in terms ____.
- **Examples:**

Terms of Sequence	n^{th} term	Compact Notation
$\left\{\frac{1}{2},\frac{2}{3},\frac{3}{4},\frac{4}{5},\ldots,\_\_\_\_\_\_\_\_,\ldots\right\}$	$a_n =$	$\left\{\_\_\_\_\right\}_{n=1}^{\infty}$
		$\left\{\frac{(-1)^n(n+1)}{3^n}\right\}$
	$a_n = \sqrt{n-3}, n \geq 3$	

- **Note:**
- Often, sequences do not have a simple defining equation.
  - o
  - o
  - o

- Let's consider a plot of the first 30 points in the sequence $\left\{ \frac{(-1)^n (n+1)}{3^n} \right\}$.



Note: The x - axis is n and the y - axis is a_n.

It appears that the terms of the sequence are ____________________________________.

- **Definition:** A sequence $\{a_n\}$ has **limit** L if ____.
  - o
  - o
  - o Notation:
  - o Note:

- Think back to chapter 2 when we looked at the limits of functions. How does the above definition for calculating the limit of a sequence differ from calculating the limit of a function?

- **Theorem:** If $\lim_{x \to \infty} f(x) = L$ and $f(n) = a_n$ when n is an integer, then $\lim_{n \to \infty} a_n = L$.

VERY NICE THEOREM!!

- **Question:** Why is this theorem helpful?

- **Limit Laws for Convergent Sequences**: Suppose *c* is a constant and $\{a_n\}$ and $\{b_n\}$ are convergent sequences. Then,

▪ $\lim_{n \to \infty} (a_n + b_n) =$	▪ $\lim_{n \to \infty} (\frac{a_n}{b_n}) =$
▪ $\lim_{n \to \infty} (a_n - b_n) =$	▪ $\lim_{n \to \infty} a_n{}^p =$
▪ $\lim_{n \to \infty} ca_n =$	▪ $\lim_{n \to \infty} c =$
▪ $\lim_{n \to \infty} (a_n b_n) =$	

- **Squeeze Theorem (for Sequences):**

- **Theorem:**

- **Example:** Find the limit of $\left\{ \frac{n+3}{n^2+5n+6} \right\}_{n=1}^{\infty}$ in the following ways: (1) factoring, (2) dividing by highest degree in denominator, and (3) using l'Hospital's rule.

First:

Second:

Third:

- **Reminders of Limits from Calculus I**
  - $\lim_{x \to \infty} (x^p) = \begin{cases} \\ \\ \end{cases}$
  - $\lim_{x \to \infty} \ln(x) =$
  - $\lim_{x \to \infty} e^x =$
  - $\lim_{x \to \infty} \tan^{-1}(x) =$

Note:

- **Example:** Find the limit of $\left\{\frac{\ln(n)}{n}\right\}_{n=1}^{\infty}$.

- **Example:** Find $\lim_{n\to\infty} \frac{\cos(n)}{\sqrt{n}}$.

- **Example:** Determine whether $a_n = (-1)^n$ is convergent or divergent.

- **Example:** Find $\left\{\frac{(-1)^n}{n}\right\}_{n=1}^{\infty}$ if it exists.

- **Question:** For what values of r is the sequence $\{r^n\}$ convergent?

- A sequence $\{a_n\}$ is called ____.
- A sequence $\{a_n\}$ is called ____.
- A sequence $\{a_n\}$ is called ____.
- **Example:** Is $a_n = \sqrt{n-3}, n \geq 3$ increasing, decreasing, or neither?

- **Example:** Is $\left\{\frac{5}{n+7}\right\}_{n=1}^{\infty}$ increasing, decreasing, or neither?

- A sequence $\{a_n\}$ is ____.
  It is____.
  If $\{a_n\}$ is ____.

- **Example:** Determine if $\{n^2\}$ is bounded below, bounded above, both, or neither.

- **Example:** Determine if $\{(-1)^n n^2\}$ is bounded below, bounded above, both, or neither.

- **Example:** Determine if the sequence $a_n = \{\cos(n)\}$ is bounded below, bounded above, both, or neither.

- **Monotonic Sequence Theorem:**

- **Example:** Without calculating the limit, show that $a_n = 3 + {}^1/_n$ is convergent.