Name_____ **Test 3- MA 107-003 Spring 2009, Mar 20 RowLetter____**

(Put your name on both the problem sheet and your answer sheets)

Answer following questions. Show **ALL of your work** to get full credit.

1. Let $f(x) = 2x^2 - 2x, g(x) = 4x + 1$.Find following composite functions:
(a) $(f \circ g)(0)$;
(b) $(g \circ f)(x)$.
(c) What's the domain of $(g \circ f)(x)$?

2. Function $f(x) = \frac{2x+1}{x-3}$ is a one-to-one function. Find its inverse f^{-1}. What's the domain of f^{-1}?

3. Write following expression as a single logarithm:

$$\log_a x + 2\log_a(x+1) - \log_a 5.$$

4. Solve following equations:
(a) $e^{x^2} = (e^x)^3 \cdot \frac{1}{e^2}$;
(b) $\ln x + \ln(x+1) = \ln(x+9)$;
(c) $\log_2(x) + 2\log_2(3) = 1$;
(d) $3^{x-1} = 5^{2x}$.

5. The normal healing of wounds can be modeled by an exponential function. If A_0 represents the original area of the wound and if A equals the area of the wound, then the formula below describes the area of a wound after n days following an injury when no infection is present to retard the healing.

$$A = A_0 e^{-0.2n}$$

Suppose that a wound initially had an area of 100 square millimeters. If healing is taking place, after how many days will the wound be one-half its original size?

Bonus: Find the value of a such that the graph of $f(x) = \log_a x$ contains points (9,2).

1. $g(0) = 4 \cdot 0 + 1 = 1$

(a) $(f \circ g)(0) = f(g(0)) = f(1) = 2 \cdot 1^2 - 2 \cdot 1 = 0$

(b) $(g \circ f)(x) = g(f(x)) = 4(f(x)) + 1 = 4 \cdot (2x^2 - 2x) + 1$.

(c) Domain of $(g \circ f)(x)$ is $\mathbb{R}$.

2. Let $y = \frac{2x+1}{x-3}$. We interchange x and y and get

$$x = \frac{2y+1}{y-3}$$

Solve for y.

$$x(y-3) = 2y+1$$
$$xy - 3x = 2y + 1$$
$$xy - 2y = 1 + 3x$$
$$(x-2)y = 1 + 3x$$
$$y = \frac{1+3x}{x-2}$$

$f^{-1}(x) = \frac{1+3x}{x-2}$. Its domain is the set of real #s $x \neq 2$.

3. $\log_a x + 2\log_a(x+1) - \log_a 5$

$= \log_a x + \log_a (x+1)^2 - \log_a 5$

$= \log_a [x(x+1)^2] - \log_a 5$

$= \log_a \frac{x(x+1)^2}{5}$

4. (a) $e^{x^2} = (e^x)^3 \cdot \frac{1}{e^2}$

$e^{x^2} = e^{3x} \cdot e^{-2}$

$e^{x^2} = e^{3x-2}$

$x^2 = 3x-2 \Leftrightarrow x^2-3x+2=0$

$(x-2)(x-1)=0$

$x=2$ or $x=1$.

(b) $\ln x + \ln(x+1) = \ln(x+9)$

$\ln[x(x+1)] = \ln(x+9)$

$x(x+1) = x+9$

$x^2+x = x+9$

$x^2 = 9$

$x=-3$ or $x=3$

check: $x=-3$: $x=-3<0$ ✗

$x=3$: $x=3>0$, $x+1=3+1>0$, $x+9=3+9>0$ ✓

Solution: $x=3$.

(c) $\log_2(x) + 2\log_2(3) = 1$

$\log_2(x) + \log_2(3^2) = 1$

$\log_2(x \cdot 3^2) = 1$

$x \cdot 3^2 = 2^1$

$9x = 2$

$x = \frac{2}{9}$

check: $x=\frac{2}{9}>0$ ✓ $\Rightarrow x=\frac{2}{9}$.

4. (c1). $3^{x-1} = 5^{2x}$

$\ln 3^{x-1} = \ln 5^{2x}$

$(x-1)\ln 3 = 2x \cdot \ln 5$

$x\cdot\ln 3 - \ln 3 = x\cdot(2\ln 5)$

$x\ln 3 - x\cdot(2\ln 5) = \ln 3$

$x(\ln 3 - 2\ln 5) = \ln 3$

$$x = \frac{\ln 3}{\ln 3 - 2\ln 5}$$

(5) $A = \frac{1}{2}A_0$

$\frac{\frac{1}{2}A_0}{A_0} = \frac{A_0 e^{-.2n}}{A_0}$

$\frac{1}{2} = e^{-.2n}$

$-.2n = \ln\frac{1}{2}$

$$n = \frac{\ln\frac{1}{2}}{-.2}$$

Bonus: $f(x) = \log_a x$ contains point $(9, 2)$.

$f(9) = 2$

$\log_a 9 = 2$

$a^2 = 9$

$a = -3$ or $a = 3$

$a > 0 \Rightarrow a = 3$.