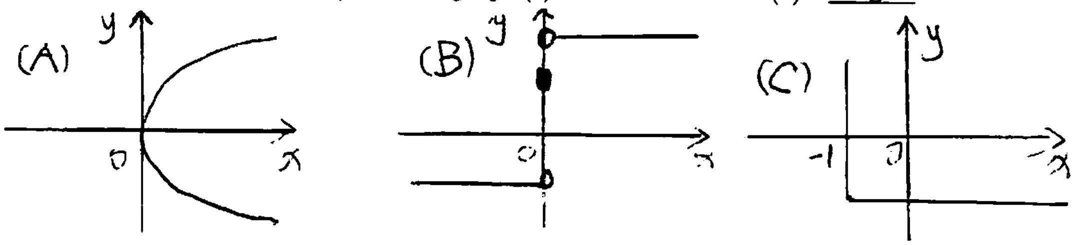
Name Key Test 1- MA 107-003 Spring 2009, Feb 2nd RowLetter____

1. Answer following questions:

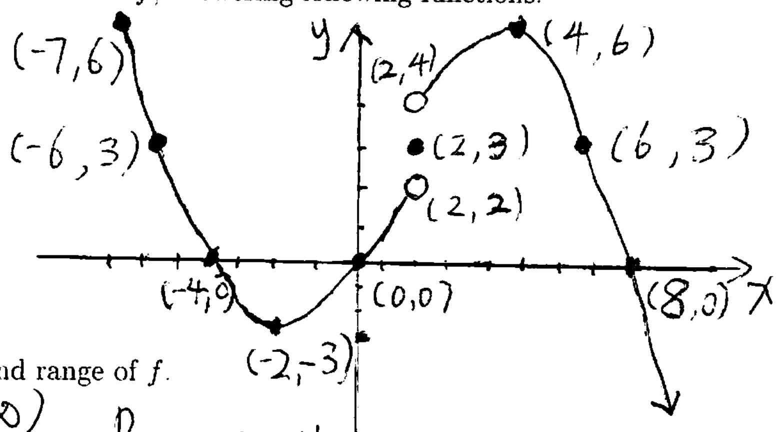
(a) Is $x = \sqrt[3]{y}$ a function? Yes

(b) Is the relation $\{(2.3), (3,5), (4,2), (4,5), (1,5)\}$ a function? No

(c) Which of following graph is/are the graph(s) of some function(s)? B



2. Given the graph of a function f, answering following functions:



(a) Find the domain and range of f.

Domain $[-3, \infty)$ Range: $y \leq 6$ or $(-\infty, 6]$

(b) Find $f(2)$.

$f(2) = 3$

(c) For what value(s) of x is $f(x) = 3$.

$x = -6, 2, 6$

(d) Find all x-intercept(s) and y-intercept(s).

x-int: $x = -4, 0, 8$

y-int: $y = 0$

(e) Find the interval(s) on which f is increasing.

$(-2, 2)$ and $(2, 4)$

(f) Find the interval(s) on which f is decreasing.

$(-7, -2)$ and $(4, \infty)$

(g) Find all local max/min. Point out at which point(s) f reaches its max/min.

max: $x = 4$, $f_{max} = 6$

min: $x = -2$, $f_{min} = -3$

(i) Find the average rate of change of f from 0 to 6.

$$\frac{\Delta y}{\Delta x} = \frac{f(6) - f(0)}{6 - 0} = \frac{3 - 0}{6} = \frac{1}{2}$$

3. Find the difference quotient of function $f(x) = \frac{1}{x}$.

$\frac{f(x+h)-f(x)}{h} =$

$$= \frac{\frac{-h}{(x+h)x}}{h}$$

$$= \frac{-h}{(x+h)\cdot x} \cdot \frac{1}{h} = \frac{-1}{(x+h)\cdot x}$$

$$f(x+h) = \frac{1}{x+h}$$

$$f(x+h) - f(x)$$

$$= \frac{1}{x+h} - \frac{1}{x}$$

$$= \frac{1\cdot x}{(x+h)x} - \frac{1\,(x+h)}{x\cdot(x+h)}$$

$$= \frac{x-(x+h)}{(x+h)x}$$

$$= \frac{x-x-h}{(x+h)x} = \frac{-h}{(x+h)x}$$

4. Let $f(x) = x^2 + 3x$.

(a) Find the slope of the secant line from $(x, f(x))$ to $(x+h, f(x+h))$ and simplify.

$$m_{sec} = \frac{f(x+h)-f(x)}{h}$$

$$= \frac{2xh+h^2+3h}{h}$$

$$= \frac{h(2x+h+3)}{h} = 2x+h+3$$

$$f(x+h) = (x+h)^2 + 3(x+h)$$

$$= x^2+2xh+h^2+3x+3h$$

$$f(x+h) - f(x)$$

$$= x^2+2xh+h^2+3x+3h-(x^2+3x)$$

$$= x^2+2xh+h^2+3x+3h-x^2-3x$$

$$= 2xh+h^2+3h$$

(b) Let $x = 1, h = 1$. Find the equation of the secant line. You needn't simplify.

$$m_{sec} = 2x+h+3 = 2\cdot1+1+3 = 6$$

use point $(x, f(x)) = (1, f(1)) = (1, 4)$ $(f(1)=4)$

$$y-4 = 6(x-1)$$

5. A trucking company transports goods between LA and Raleigh, a distance of 2240 miles. The company's policy is to charge, for each pound, \$0.60 per mile for the first 500 miles, \$0.50 per mile for the next 700 miles, \$2.00 per mile for the next 800 miles, and no charge for the remaining 240 miles.

(a) Express the cost C as a function of mileage x between 500 miles and 1200 miles from LA.

C = charge for first 500 miles + charge for remaining $(x-500)$ miles

$$C = .60 \cdot 500 + .50 \cdot (x-500)$$

(b) Express the cost C as a function of mileage x between 1200 miles and 2000 miles from LA.

$$C = .60 \cdot 500 + .50 \cdot 700 + 2 \cdot (x-1200)$$

(c) Express the cost C as a function of mileage x between 2000 miles and 2240 miles from LA.

$$C = .60 \cdot 500 + .50 \cdot 700 + 2 \cdot 800$$

6. Find transformations applied to the graph of $y = \sqrt{x}$ in order to get the graph of $y = 4\sqrt{2x+3} - 1$.

$$y=\sqrt{x} \xrightarrow{1} y=\sqrt{x+3} \xrightarrow{2} y=\sqrt{2x+3} \xrightarrow{3} y=4\sqrt{2x+3} \xrightarrow{4} y=4\sqrt{2x+3}-1$$

1: $f(x) \to f(x+3)$: Shift Left 3 units

2: $f(x) \to f(2x)$: horizontal compress by a factor of $\frac{1}{2}$

3: $f(x) \to 4f(x)$: vertical stretch by a factor of 4

4: $f(x) \to f(x)-1$: shift down 1 unit

7. Write down the new functions you get at each step after applying given transformations to the graph of $f(x) = x^2$ in order: Don't Simplify.

(a) Shift right 2 units:

$f(x) = (x - 2)^2$

(b) Then shift up 3 units:

$f(x) = (x - 2)^2 + 3$

(c) Then reflect about y-axis:

$f(x) = (-x - 2)^2 + 3$

(d) Finally stretch vertically by a factor of 4:

$f(x) = 4[(-x - 2)^2 + 3]$

8. Factor following expression

$2(x^2 + 6x + 1)(x + 3)^2 + 2(x^2 + 6x + 1)^2$

$= 2(x^2+6x+1)[(x+3)^2 + (x^2+6x+1)]$

$= 2(x^2+6x+1)[x^2+6x+9+x^2+6x+1]$

$= 2(x^2+6x+1)(2x^2+12x+10)$

$= 2(x^2+6x+1)\,2(x^2+6x+5)$

$= 4(x^2+6x+1)(x+1)(x+5)$

$((x^2+6x+5) = (x+1)(x+5))$