Name key Test 3- MA 107-002 SS1 2009, Jun 17th Grade________

(Write down **ALL** of your work to get full credit. Good luck!)

1. How long does it take for an investment to triple in value if it is invested at 10% per annum compounded monthly?

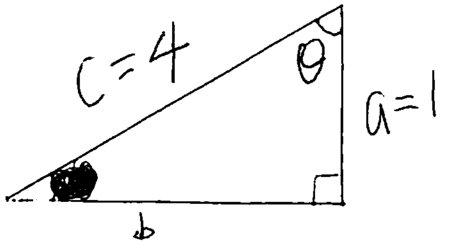
2. Suppose the half-life of some radioactive substance is 200 years. Find the decay rate of this radioactive substance. How long does it take to become one fourth of the original amount of the substance? (Hint: it follows the exponential law)

3. Find the region bounded by following curves (sketch curves and shade the bounded region) and identify all vertices:

$$y = 1, \quad y - 2x = 1, \quad y = 5 - 2x.$$

4. Convert following angles between degrees and radians:
(a) 40°; (b) $-\frac{\pi}{18}$.

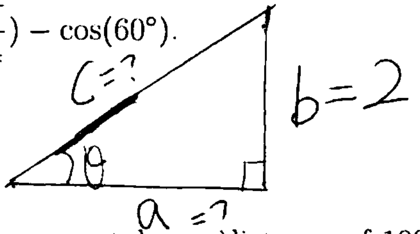
5. Find all six trig functions of the angle $\theta$ for the following right triangle:



6. Find the exact value of following expression:

$$\sin^2(60°) + \cos^2(\frac{\pi}{4}) - \cos(60°).$$

7. In the given right trianle, $\sin(\theta) = \frac{1}{3}$ and $b = 2$.
(a) Find $a$ and $c$.
(b) Find values of $\cot(\theta)$ and $\sec(\theta)$.



8. To measure the height of a building, two sightings are taken a distance of 100 feet apart. If the first angle of elevation is 42° and the second is 30°, what is the height of the building?

**Bonus:** Let $(-1, \sqrt{3})$ be a point on the terminal side of some angle $\theta$. Here $\theta$ is some general angel and actually not an acute one. Find all six trig functions of this angle $\theta$.

$$r = \sqrt{(-1)^2 + (\sqrt{3})^2} = \sqrt{1+3} = 2$$

$$\sin\theta = \frac{y\text{-comp}}{r} = \frac{\sqrt{3}}{2} \qquad \cos\theta = \frac{x\text{-comp}}{r} = \frac{-1}{2} = -\frac{1}{2}$$

$$\tan\theta = \frac{y\text{-comp}}{x\text{-comp}} = \frac{\sqrt{3}}{-1} = -\sqrt{3} \qquad \cot\theta = \frac{x\text{-comp}}{y\text{-comp}} = \frac{-1}{\sqrt{3}} = \frac{-1\cdot\sqrt{3}}{\sqrt{3}\cdot\sqrt{3}} = -\frac{\sqrt{3}}{3}$$

$$\sec\theta = \frac{r}{x\text{-comp}} = \frac{2}{-1} = -2 \qquad \csc\theta = \frac{r}{y\text{-comp}} = \frac{2}{\sqrt{3}} = \frac{2\cdot\sqrt{3}}{\sqrt{3}\cdot\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

# Formulas you might use

$$\tan(\theta)\cdot\cot(\theta) = 1 \ (\tan(\theta) = \frac{1}{\cot(\theta)},\ \cot(\theta) = \frac{1}{\tan(\theta)})$$

$$\sec(\theta)\cdot\cos(\theta) = 1 \ (\sec(\theta) = \frac{1}{\cos(\theta)},\ \cos(\theta) = \frac{1}{\sec(\theta)})$$

$$\csc(\theta)\cdot\sin(\theta) = 1 \ (\csc(\theta) = \frac{1}{\sin(\theta)},\ \sin(\theta) = \frac{1}{\csc(\theta)})$$

$$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)};\ \cot(\theta) = \frac{\cos(\theta)}{\sin(\theta)}$$

$$\sin^2(\theta) + \cos^2(\theta) = 1$$

$$\tan^2(\theta) + 1 = \sec^2(\theta)$$

$$\cot^2(\theta) + 1 = \csc^2(\theta)$$

key

1. $A = P(1+\frac{r}{n})^{nt}$

$A = 3P$ , $10\% = .1$

$\frac{3P}{P} = \frac{P}{P}(1+\frac{0.1}{12})^{12t}$

$3 = (1+\frac{1}{120})^{12t}$

$3 = (\frac{121}{120})^{12t}$

$\ln 3 = 12t \ln \frac{121}{120}$

$t = \frac{\ln 3}{12 \cdot \ln \frac{121}{120}}$.

2. Half-life is 200 years $\Rightarrow A(200) = \frac{1}{2}A_0$

$A(t) = A_0 e^{kt}$

So $\frac{\frac{1}{2}A_0}{A_0} = \frac{A_0}{A_0} e^{k \cdot 200}$

$\frac{1}{2} = e^{200k}$

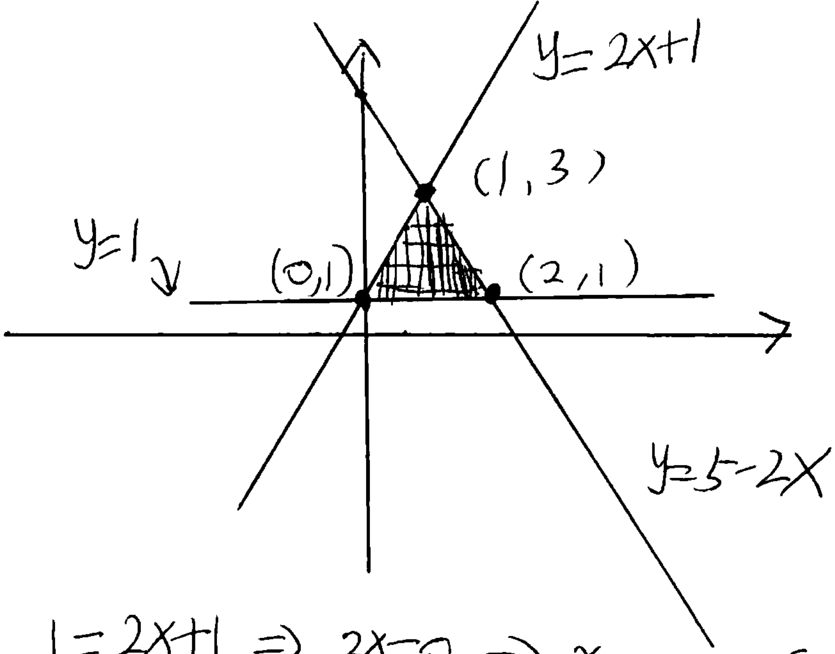
$200k = \ln\frac{1}{2} \Rightarrow k = \frac{\ln\frac{1}{2}}{200}$.

$\frac{1}{4}A = \frac{1}{2}(\frac{1}{2}A)$

$A_0 \xrightarrow{200} \frac{1}{2}A_0 \xrightarrow{200} \frac{1}{2}(\frac{1}{2}A_0)$

So it takes 400 years to become $\frac{1}{4}$ of the original amount.

3, $y-2x=1 \Leftrightarrow y=2x+1$



$\begin{cases} y=1 \\ y=2x+1 \end{cases} \Rightarrow 1=2x+1 \Rightarrow 2x=0 \Rightarrow x=0 \Rightarrow \begin{cases} x=0 \\ y=1 \end{cases} \Rightarrow (0,1)$

$\begin{cases} y=1 \\ y=5-2x \end{cases} \Rightarrow 1=5-2x \Rightarrow 2x=4 \Rightarrow x=2 \Rightarrow \begin{cases} x=2 \\ y=1 \end{cases} \Rightarrow (2,1)$

$\begin{cases} y=2x+1 \\ y=5-2x \end{cases} \Rightarrow 2x+1=5-2x \Rightarrow 2x+2x=5-1$

$4x=4$

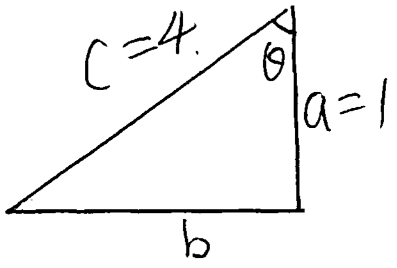
$x=1 \Rightarrow \begin{cases} x=1 \\ y=2*1+1=3 \end{cases}$

$\Rightarrow (1,3)$

4. (a): $40^\circ = 40 * 1^\circ = 40 * \frac{\pi}{180} = \frac{40}{180}\pi = \frac{4}{18}\pi = \boxed{\frac{2}{9}\pi \text{ radian}}$ ③

(b) $-\frac{\pi}{18} = -\frac{\pi}{18} * 1 = -\frac{\pi}{18} * \frac{180^\circ}{\pi} = -\frac{180^\circ}{18} = -10^\circ.$

5.



$a^2+b^2=c^2$

$1^2+b^2=4^2$

$b^2=16-1$

$b^2=15$

$b=-\sqrt{15}$ ✗ or $b=\sqrt{15}$.

According to definition

$\sin\theta = \frac{O}{H} = \frac{\sqrt{15}}{4}$

$\cos\theta = \frac{A}{H} = \frac{1}{4}$

$\tan\theta = \frac{O}{A} = \frac{\sqrt{15}}{1} = \sqrt{15}$

$\cot\theta = \frac{A}{O} = \frac{1}{\sqrt{15}} = \frac{1\cdot\sqrt{15}}{\sqrt{15}\cdot\sqrt{15}} = \frac{\sqrt{15}}{15}$

$\sec\theta = \frac{H}{A} = \frac{4}{1} = 4$

$\csc\theta = \frac{H}{O} = \frac{4}{\sqrt{15}} = \frac{4\cdot\sqrt{15}}{\sqrt{15}\cdot\sqrt{15}} = \frac{4\sqrt{15}}{15}.$

6. $\sin^2(60^\circ) + \cos^2(\frac{\pi}{4}) - \cos(60^\circ)$

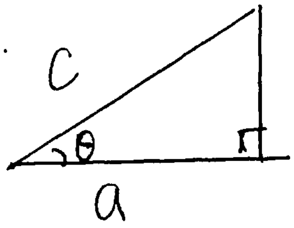
$= (\frac{\sqrt{3}}{2})^2 + (\frac{\sqrt{2}}{2})^2 - \frac{1}{2}$

$= \frac{3}{4} + \frac{2}{4} - \frac{1}{2}$

$= \frac{3}{4} + \cancel{\frac{1}{2}} - \cancel{\frac{1}{2}}$

$= \frac{3}{4}$

7.



$\sin\theta = \frac{1}{3}$

$$\sin\theta = \frac{b}{c} \Rightarrow \frac{1}{3} = \frac{2}{c} \Leftrightarrow \cancel{3}C\cdot\frac{1}{\cancel{3}} = \frac{2}{\cancel{c}}\cdot 3\cdot\cancel{c}$$

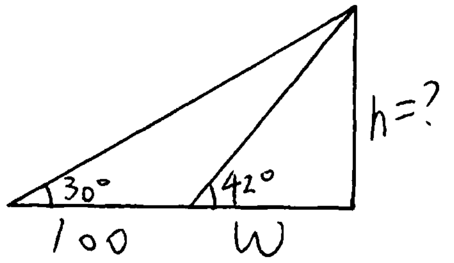
$$C = 2\cdot 3 = 6$$

$$a = \sqrt{c^2-b^2} = \sqrt{6^2-2^2} = \sqrt{32} = 4\sqrt{2}$$

$$\cot\theta = \frac{A}{O} = \frac{4\sqrt{2}}{2} = 2\sqrt{2}$$

$$\sec\theta = \frac{H}{A} = \frac{6}{\cancel{2}\sqrt{2}} = \frac{3}{2\sqrt{2}} = \frac{3\sqrt{2}}{2\sqrt{2}\cdot\sqrt{2}} = \frac{3\sqrt{2}}{4}$$

8.



$$\begin{aligned} 42°&: \tan 42° = \frac{h}{w} \\ 30°&: \tan 30° = \frac{h}{w+100} \end{aligned} \Rightarrow \begin{cases} h = w\cdot\tan 42° \\ h = (w+100)\tan 30° \end{cases}$$

$$\Rightarrow w\cdot\tan 42° = (w+100)\tan 30°$$

$$w\cdot\tan 42° = w\cdot\tan 30° + 100 * \tan 30°$$

$$w\cdot\tan 42° - w\cdot\tan 30° = 100 * \tan 30°$$

$$w(\tan 42° - \tan 30°) = 100 * \tan 30°$$

$$w = \frac{100\cdot\tan 30°}{\tan 42° - \tan 30°}$$

$$\Rightarrow h = w\cdot\tan 42° = \frac{100\cdot\tan 30°\cdot\tan 42°}{\tan 42° - \tan 30°}.$$