Name key   Test 2- MA 107-003 Spring 2009, Feb 23   RowLetter____

(20) 1. Let $f(x) = -2x^2 + 5x - 3$ and its graph is a parabola. Answer following questions:

(3) (a) It opens up or down?

down

(5) (b) Find its x-intercept(s) and y-intercept.

x-int: $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-5 \pm \sqrt{5^2-4\cdot(-2)(-3)}}{2\cdot(-2)} = \frac{-5\pm1}{-4}$

$\Rightarrow x_1 = \frac{-6}{-4} = \frac{3}{2}$. $(\frac{3}{2}, 0)$ $x_2 = \frac{-4}{-4} = 1$. $(1,0)$

y-int: $x=0 \Rightarrow f(0) = -3$. $(0,-3)$

(4) (c) Find its vertex (h, k).

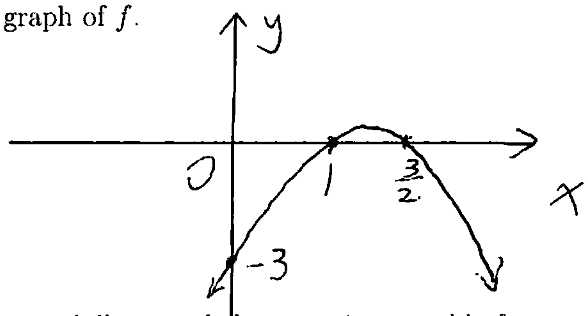
$h = \frac{-b}{2a} = \frac{-5}{2(-2)} = \frac{5}{4}$

$k = \frac{4ac-b^2}{4a} = \frac{4\cdot(-2)(-3)-5^2}{4\cdot(-2)} = \frac{1}{8}$ $\Rightarrow (h,k) = (\frac{5}{4}, \frac{1}{8})$

(4) (d) f has a local max or min? What's its max/min? Point out where it reaches its max/min.

$a<0 \Rightarrow f$ has a local max. $f_{max} = k = \frac{1}{8}$ at $x = h = \frac{5}{4}$

(4) (e) Draw the graph of f.



(15) 2. The price, p, in dollars, and the quantity, x, sold of a certain product obey the demand equation below:

$$p = -2x + 400, \ 0 \le x \le 200.$$

(4) (a) Express the revenue R as a function of x.

$R = p\cdot x = (-2x+400)\cdot x = -2x^2 + 400x$

(4) (b) What is the revenue if 50 units are sold?

$R(50) = (-2\cdot 50 + 400)\cdot 50 = 300\cdot 50 = 15,000$

(4) (c) What quantity x maximizes revenue? What is the maximum revenue?

$x = \frac{-b}{2a} = \frac{-400}{2(-2)} = 100$

$R_{max} = k = \frac{4ac-b^2}{4a} = \frac{4\cdot(-2)\cdot 0 - 400^2}{4\cdot(-2)} = 20,000$

(3) (d) What price should the company charge to maximize revenue?

$p = -2x + 400 = -2\cdot 100 + 400 = 200$

(20) 3. Let $f(x) = 2(x-3)^2(x-4)^3$.

(a) For large $|x|$, the graph of $f(x)$ resembles the graph of which power function $g(x)$?

(4) $n = 2+3 = 5$, $a_3 = 2 \cdot 1 \cdot 1 = 2$

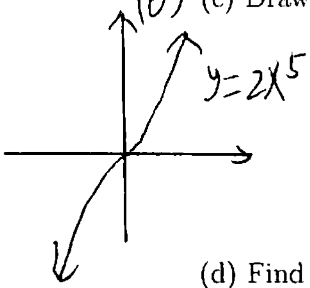
$g(x) = 2x^5$

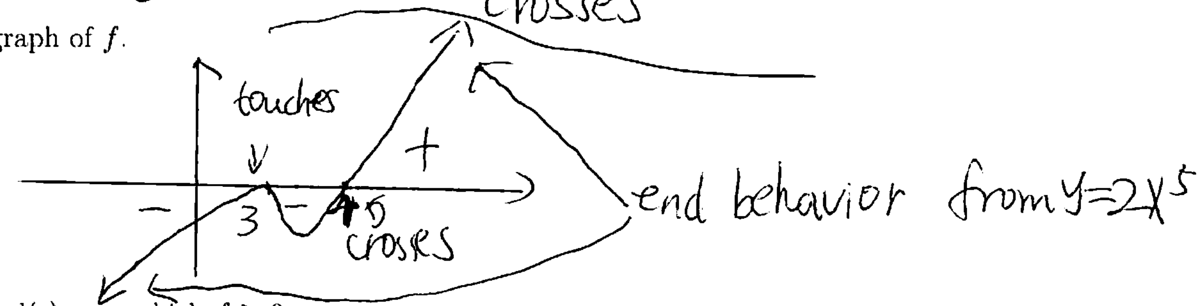
(b) Find zeros of f and their multiplicities respectively. Tell if they touch/cross the x-axis.

(6)

zeros	multi	touches/crosses
$x=3$	2	touches
$x=4$	3	crosses

(6) (c) Draw the graph of f.





(d) Find interval(s) over which $f \geq 0$.

(4) $[4, \infty)$ or $x \geq 4$

(10) 4. Let $R(X) = \frac{2x(x-4)(x+5)}{(x+3)^2(x-4)}$.

(a) Find the domain of $R(x)$.

(3) ~~$x \neq$~~ all real numbers except $x=-3$ and $x=4$

($x \neq -3$ and $x \neq 4$)

(b) Find the vertical asymptote(s) of $R(x)$.

(3) lines $x = -3$ and $x = 4$

(c) Find the horizontal asymptote(s) of $R(x)$.

(4) degree $(P) = 3$ = degree (q) $\left(R(x) = \frac{P(x)}{q(x)}\right)$

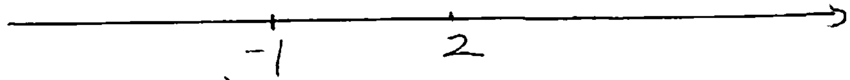
$\Rightarrow$ HA: $y = \frac{\text{Leading coef of } P}{\text{Leading coef of } q} = \frac{2}{1} = 2$.

$\boxed{y=2}$

(15) 5. Solve inequality $\frac{2x-4}{(x+1)^2} \geq 0$.

zeros: $2x-4=0 \Rightarrow 2x=4 \Rightarrow x=2$

Domain: $(x+1)^2 \neq 0 \Rightarrow x+1 \neq 0 \Rightarrow x \neq -1$.



$(-\infty, -1)$ $+$ $-$ $x=-2$, $f(-2)<0$

$(-1, 2)$ $+$ $-$ $x=0$, $f(0)=\frac{-4}{1}<0$

$(2, \infty)$ $+$ $-$ $x=3$ $f(3)>0$

$f(x)=\frac{2x-4}{(x+1)^2}>0 \Rightarrow x>2$

$\Rightarrow f(x)=\frac{2x-4}{(x+1)^2} \geq 0 \Rightarrow x \geq 2$. ($x=2$ is a zero of $f(x)$)

(10) 6. Let $f(x) = x^3 + x - 3$ be divided by $g(x) = x + 2$. Find the quotient $q(x)$ and the remainder $r(x)$ such that $f(x) = q(x)g(x) + r(x)$, where either $r(x) = 0$ or the degree of $r(x)$ is less than that of $g(x)$.

$$
\begin{array}{r}
x^2 - 2x + 5 \leftarrow q(x) \\
x+2 \overline{) x^3 + 0x^2 + x - 3} \\
-(x^3 + 2x^2) \\
\hline
-2x^2 + x \\
-(-2x^2 - 4x) \\
\hline
5x - 3 \\
-(5x + 10) \\
\hline
-13 \leftarrow r(x)
\end{array}
$$

So $q(x) = x^2 - 2x + 5$

$r(x) = -13$.

(10) 7. Let $f(x) = x^3 + 4x + 5$.

(5) (a) Find all possible rational zeros f could have.

Factors of $a_3 = 1$: $q = \pm 1$

Factors of $a_0 = 5$: $p = \pm 1, \pm 5$

$\frac{p}{q} = \pm 1, \pm 5$.

(5) (b) Use part (a) to find out all rational zeros of f.

$f(1) \neq 0$

$f(-1) = 0$

$f(5) \neq 0$

$f(-5) \neq 0$

$\Rightarrow$ $x = -1$ is the only rational zero of f.

(plug $\frac{p}{q}$ into the definition of f to get the values of f at those points)

(+2) **Bonus.** Let $f(x)$ be a complex polynomial of degree 5 with real coefficients. We know it has three zeros: $1, 3i, 2+i$. Find the rest of its zeros.

$1, 3i, 2+i, \overline{3i}, \overline{2+i}$

$\overline{3i} = -3i$, $\overline{2+i} = 2-i$.

So the rest are $-3i, 2-i$.