Name Key ________________ **Test 2- MA 107-002 SS1 2009, Jun 9th** Grade_________

(Write down **ALL** of your work to get full credit. Good luck!)

1. Let $f(x) = (x-5)^2(2x-4)^3$.

(a) Find the degree n and leading coefficient a_n of f. Then for large $|x|$, the graph of $f(x)$ resembles the graph of which power function $g(x)$?

(b) Find zeros of f and their multiplicities respectively. Tell if they touch/cross the x-axis.

(c) Draw the graph of f.

(d) Find interval(s) over which $f > 0$.

2. Let $R(X) = \frac{2x(x+4)(x-5)}{3(x-3)^2(x+4)}$.

(a) Find the domain of $R(x)$.

(b) Find the vertical asymptote(s) of $R(x)$ if any.

(c) Find the horizontal asymptote(s) of $R(x)$ if any.

3. Solve inequality $\frac{x+1}{2x-4} \geq 0$ algebraically.

4. Let $f(x) = x^3 - 3x^2 - 1$ be divided by $g(x) = x - 1$. Find the quotient $q(x)$ and the remainder $r(x)$ such that $f(x) = q(x)g(x) + r(x)$, where either $r(x) = 0$ or the degree of $r(x)$ is less than that of $g(x)$.

5. Let $f(x) = x^3 + 2x + 3$.

(a) Find all possible rational zeros f could have.

(b) Use part (a) to find out all rational zeros of f.

6. Function $f(x) = \frac{x+1}{x-3}$ is a one-to-one function. Find its inverse f^{-1}. You don't need to check $(f \circ f^{-1})(x) = x$ and $(f^{-1} \circ f)(x) = x$ here.

7. Solve following equations: (don't forget to check your answer if it's a log equation)

(a) $e^{x^2} = (e^x)^2 \cdot \frac{1}{e}$;

(b) $\ln x + \ln(x+1) = \ln(x+4)$;

(c) $\log_2(x) + 2\log_2(3) = 2$;

(d) $3^{x-1} = 7^{3x}$.

8. The normal healing of wounds can be modeled by an exponential function. If A_0 represents the original area of the wound and if A equals the area of the wound, then the formula below describes the area of a wound after n days following an injury when no infection is present to retard the healing.

$$A = A_0 e^{-0.3n}$$

Suppose that a wound initially had an area of 200 square millimeters. If healing is taking place, after how many days will the wound be one-half its original size?

1. $n = 2 + 3 = 5$

(a) $a_5 = 1^2 \cdot 2^3 = 8$ $\Big\} \Rightarrow g(x) = 8 \cdot x^5$

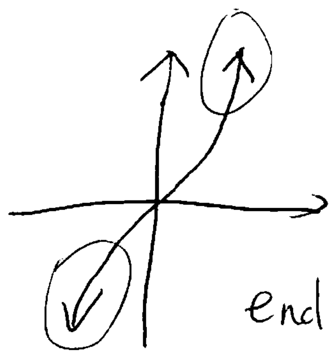
(b) zeros: $x - 5 = 0$ or $2x - 4 = 0$

$x = 5$; $2x = 4$, $x = 2$

multiplicity: 2 — even — touches

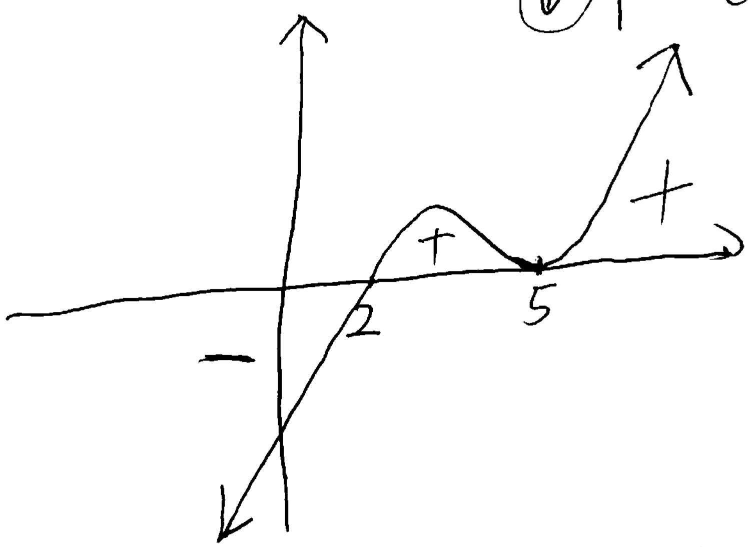
multiplicity 3 — odd — crosses.

(c) $g(x) = 8 \cdot x^5$:



(look like $y = x^3$)

end behavior



(d). $f(x) > 0 \Rightarrow 2 < x < 5$ or $x > 5$.

(Note: $f(x) \neq 0 \Rightarrow x \neq 2$ or 5, so you shoud not include the point $x = 5$ in your answer)

2. $R(x) = \frac{p(x)}{q(x)}$

(a) $q(x) = 0 \Rightarrow \quad x = 3$ or $x = -4$

$\Rightarrow$ Domain: all real numbers except 3 and -4.

(b) To find VA, you need to make sure $R(x)$ in the lowest terms first, ie. Cancel all common factors of p and q.

$$R(x) = \frac{2x\cancel{(x+4)}(x-5)}{3(x-3)^2\cancel{(x+4)}} = \frac{2x(x-5)}{3(x-3)^2}$$

$\Rightarrow$ VA: $x = 3$.

(c) $n = \text{degree}(p(x)) = 1+1+1 = 3$

$m = \text{degree}(q(x)) = 2+1 = 3$

$n = m \Rightarrow$ HA: $y = \frac{\text{L.C. of } p}{\text{L.C. of } q} = \frac{2}{3}$

or $y = \frac{2}{3}$.

3. $f(x) = \dfrac{x+1}{2x-4}$ ③

Find zeros of f and points excluded from the domain of f:

$f(x) = 0 \Rightarrow \quad x+1=0 \quad \& \quad 2x-4 \neq 0$

$x = -1 \qquad\qquad 2x \neq 4$

$x \neq 2$

$\Rightarrow$

$x=-2 \in (-\infty, -1) \; + \; - \quad f(-2) = \dfrac{-2+1}{2\cdot(-2)-4} = \dfrac{-1}{-8} = \dfrac{1}{8} > 0$

$x=0 \in (-1, 2) \; + \; - \quad f(0) = \dfrac{0+1}{2\cdot 0-4} = \dfrac{1}{-4} = -\dfrac{1}{4} < 0$

$x=3 \in (2, \infty) \; + \; - \quad f(3) = \dfrac{3+1}{2\cdot 3-4} = \dfrac{4}{2} > 0$

$f(x) > 0 \Rightarrow \quad x < -1 \quad \text{or} \quad x > 2$

$f(x) \geq 0 \Rightarrow \quad x \leq -1 \quad \text{or} \quad x > 2.$

4. Do Long division.

$$x^2 \quad -2x \quad -2 \quad \leftarrow q(x)$$

$$x-1 \overline{) \, x^3 - 3x^2 + 0x - 1}$$

$$-(x^3 - x^2)$$

$$-2x^2 + 0x$$

$$-(-2x^2 + 2x)$$

$$-2x \quad -1$$

$$-2x \quad +2$$

$$-3 \quad \leftarrow r(x)$$

So $q(x) = x^2 - 2x - 3$

$r(x) = -3$

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5. zeros of f have the form $\frac{p}{q}$. p is from $a_0 = 3$. ⑤

(a) q is from $a_n = 1$. so

$$\left.\begin{array}{l} p = \pm 1, \pm 3 \\ q = \pm 1 \end{array}\right\} \Rightarrow \frac{p}{q} = \pm 1, \pm 3$$

(b) $f(1) = 1^3 + 2 \cdot 1 + 3 = 6 \neq 0$ ✗

$f(-1) = (-1)^3 + 2 \cdot (-1) + 3 = -1 - 2 + 3 = 0$ ✓

$f(3) = 3^3 + 2 \cdot 3 + 3 \neq 0$ ✗

$f(-3) = (-3)^3 + 2(-3) + 3 = -27 - 6 + 3 \neq 0$ ✗

(6) So the only rational zero is $x = -1$.

6. $y = \frac{x+1}{x-3} \Rightarrow x = \frac{y+1}{y-3}$

$$\Rightarrow (y-3) \cdot x = \frac{y+1}{\cancel{y-3}} \cancel{(y-3)}$$

$$yx - 3x = y + 1$$

$$yx - y = 1 + 3x$$

$$(x-1)y = 1 + 3x$$

$$y = \frac{3x+1}{x-1}$$

$$\Rightarrow f^{-1}(x) = \frac{3x+1}{x-1}$$

7. (a) $e^{x^2} = (e^x)^2 \cdot \frac{1}{e}$

$e^{x^2} = e^{2x} \cdot e^{-1}$

$e^{x^2} = e^{2x-1}$

$x^2 = 2x - 1$

$x^2 - 2x + 1 = 0$

$(x-1)^2 = 0$

$x = 1$

(b) $\ln x + \ln(x+1) = \ln(x+4)$

$\ln[x(x+1)] = \ln(x+4)$

$x(x+1) = x+4$

$x^2 + \cancel{x} = \cancel{x} + 4$

$x^2 = 4$

$x = -2$ or $x = 2$

check: $x = -2 < 0$ ✗

$x = 2 > 0$, $x+1 = 2+1 = 3 > 0$, $x+4 = 2+4 = 6 > 0$.

$\Rightarrow$ $x=2$ is the only solution.

7.(c) $\log_2(x) + 2\log_2(3) = 2$

$$\log_2(x) + \log_2(3^2) = 2$$

$$\log_2(x \cdot 3^2) = 2$$

$$x \cdot 3^2 = 2^2$$

$$9x = 4$$

$$x = \frac{4}{9}$$

check: $x = \frac{4}{9} > 0$ ✓

$\Rightarrow x = \frac{4}{9}$ is a solution.

(d) $3^{x-1} = 7^{3x}$

$$\ln 3^{x-1} = \ln 7^{3x}$$

$$(x-1)\cdot \ln 3 = 3x \cdot \ln 7$$

$$x\ln 3 - \ln 3 = x \cdot (3\ln 7)$$

$$x \cdot \ln 3 - x \cdot (3\ln 7) = \ln 3$$

$$x(\ln 3 - 3\ln 7) = \ln 3$$

$$x = \frac{\ln 3}{\ln 3 - 3\ln 7}$$

$\cancel{8.\ A = \frac{1}{2}A_0 \Rightarrow \frac{1}{2}A_0 = A_0 e^{-.03t}}$

$\cancel{\frac{1}{2} = e^{\dots}}$

8. $A=\frac{1}{2}A_0$

$$\Rightarrow \frac{\frac{1}{2}\cancel{A_0}}{\cancel{A_0}} = \frac{\cancel{A_0}e^{-.3n}}{\cancel{A_0}}$$

$$\frac{1}{2} = e^{-.3n}$$

$$-.3n = \ln\frac{1}{2}$$

$$n = \frac{\ln\frac{1}{2}}{-.3}$$