REGIONS BOUNDED BY CURVES

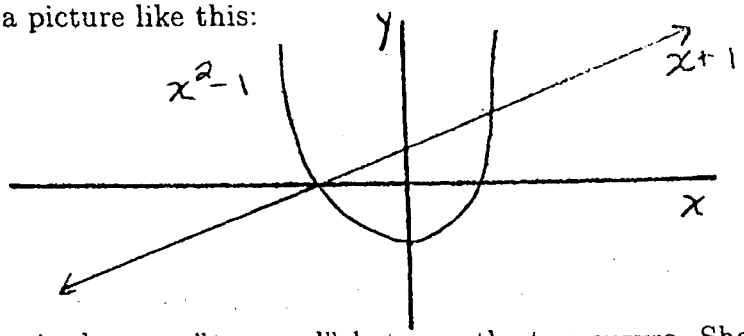
Name:

In calculus, you will be asked to find the area of a region between two curves on a graph. In this class, we will look at how to sketch the regions and find the vertices (or intersection points) of two or more functions on one graph.

Let's look at a problem: EX 1 - Sketch the following graphs on the same set of axes and shade the bounded region.

$$y = x^2 - 1, \qquad y = x + 1$$

These are both functions we know how to graph these functions. If we put both graphs on the same set of axes we get a picture like this:



The "bounded" region is the area "trapped" between the two curves. Shade what you think the bounded region would be on this graph.

Now, what we need to be able to find are the "corner points" or VERTICES of these two graphs. Note: the term VERTEX here means something different than it did before. Instead of the vertex of a parabola, we're looking for the vertices, or INTERSECTION POINTS of these graphs. To find them, we need to solve a SYSTEM OF EQUATIONS.

TO FIND THE VERTICES:

Two graphs will intersect when one x value gives the same y value for each function. So, if the y's are equal, we can set the two equations (for our example, $y = x^2 - 1$, and $y = x + 1$) equal to each other and solve for x.

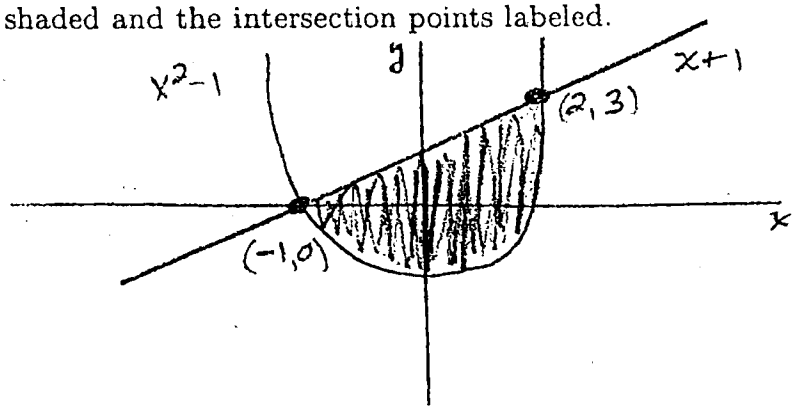
$$x^2 - 1 = x + 1$$

$$x^2 - x - 2 = 0$$

$$(x - 2)(x + 1) = 0$$

$$x = 2 \text{ or } x = -1$$

Plug these x values back into EITHER y equation to find that our two intersection points for this problem are $(2, 3)$ and $(-1, 0)$ and for our final answer we would sketch the graph as shown, with the bounded region shaded and the intersection points labeled.



NORTH CAROLINA STATE UNIVERSITY
Department of Mathematics

MA 107 Supplement -- Regions Bounded by Curves

In calculus, finding the area of a region bounded by curves is an important concept. Precalculus lays the foundation for this concept by sketching the curves, shading the bounded region and by using algebra to determine the points of intersection.

In the following problems, sketch the curves and shade the bounded region. Identify the vertices.

1. $y = x^2 - 1$ and $y = x + 1$

2. $y = x^2$, $y = 0$, $x = 1$, and $x = 3$

3. $x^2 = 4 + y$ and $x^2 + y = 6$

4. $y = 6x - x^2$ and $y = x^2 - 2x$

5. $y = x^4$ and $y = x^2$

6. $y = 3$, $y = x$, and $x = 0$

7. $y = x^2$ and $y = \sqrt{x}$

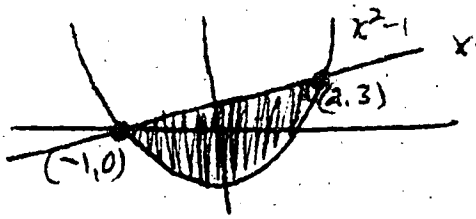
8. $y = 5$, $y = \sqrt{x}$, and $x = 0$

9. $y = 2x + 1$ and $y = x^2 + 1$

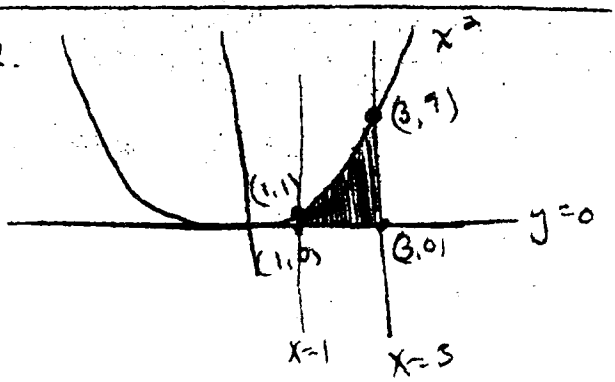
10. $x + 2y = 2$, $y - x = 1$, $2x + y = 7$

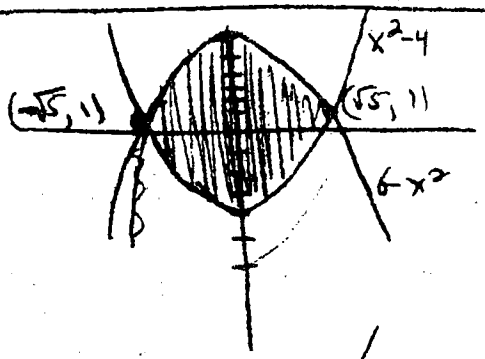
Regions Bounded by Curves — Solutions

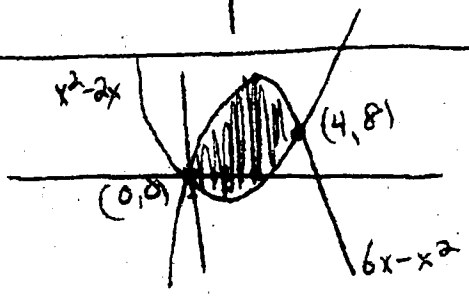
1.

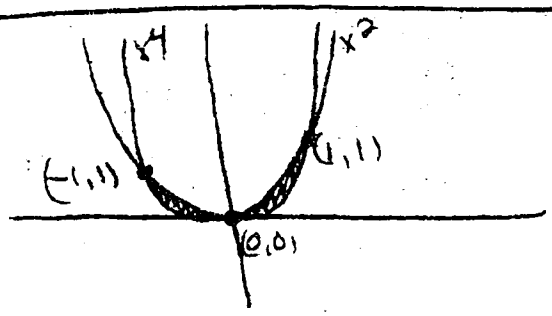


2.

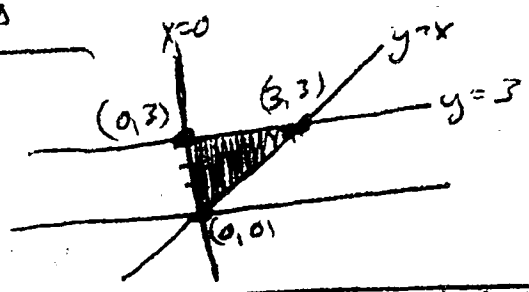




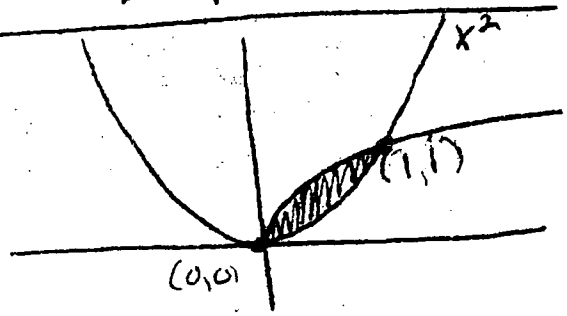




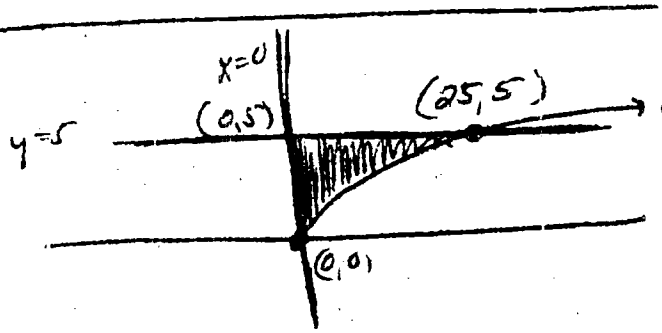
6.



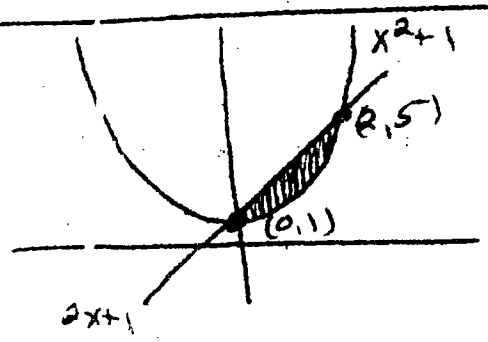
7.



8.



9.



10.