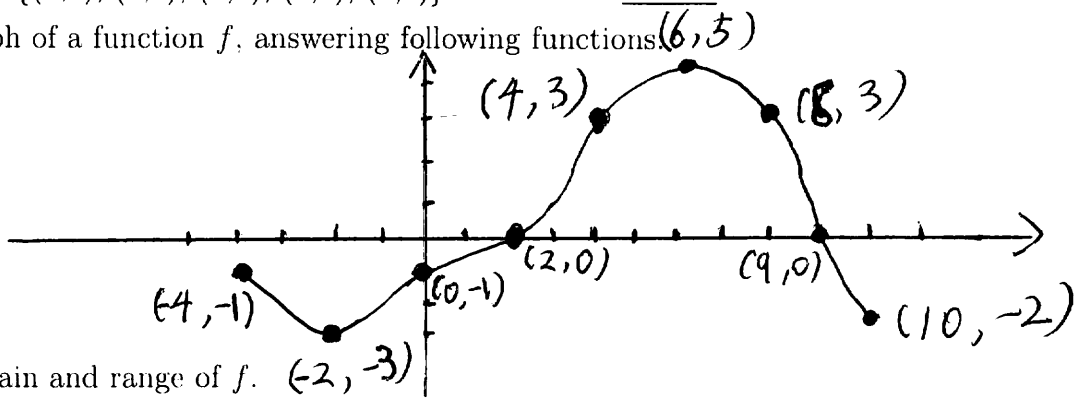
Name Key Test 1- MA 107-002 SS1 2009, May 28th Grade_____

1. Answer following questions:

(a) Is $x = \sqrt[2]{y}$ a function? _____

(b) Is the relation {(1,5), (2,5), (3,5), (4,5), (5,5)} a function? _____

2. Given the graph of a function f, answering following functions:



(a) Find the domain and range of f.

(b) Find $f(2)$.

(c) For what value(s) of x is $f(x) = 3$.

(d) Find all x-intercept(s) and y-intercept(s).

(e) Find the interval(s) on which f is increasing.

(f) Find the interval(s) on which f is decreasing.

(g) Find all local max/min if any, and point out at which point(s) f reaches its max/min.

(i) Find the average rate of change of f from 0 to 4.

(j) Find the linear equation of the secant line connecting points $(0, f(0))$ and $(4, f(4))$.

3. Let $f(x) = x^2 + 3x + 1$. Find the difference quotient of f: $\frac{f(x+h)-f(x)}{h}$, and simplify.

4. A trucking company transports goods between LA and Raleigh, a distance of 2240 miles. The company's policy is to charge, for each pound, \$0.60 per mile for the first 500 miles, \$0.50 per mile for the next 700 miles, \$2.00 per mile for the next 800 miles, and no charge for the remaining 240 miles.

(a) Express the cost C as a function of mileage x between 500 miles and 1200 miles from LA.

(b) Express the cost C as a function of mileage x between 1200 miles and 2000 miles from LA.

(c) Express the cost C as a function of mileage x between 2000 miles and 2240 miles from LA.

5. Find transformations applied to the graph of $y = \sqrt[3]{x}$ in order to get the graph of $y = 4\sqrt[3]{x+3} - 1$.

6. Write down the new functions you get at each step after applying given transformations to the graph of $f(x) = x^3$ in order:

(a) Shift right 2 units;

(b) **Then** stretch vertically by a factor of 4;

(c) **Then** reflect about x-axis.

7. The price, p, in dollars, and the quantity, x, sold of a certain product obey the demand equation below:

$$p = -2x + 40, \ 0 \leq x \leq 20.$$

(a) Express the revenue R as a function of x.

(b) What is the revenue if 5 units are sold?

(c) What quantity x maximizes revenue? What is the maximum revenue?

8. Factor following expression

$$3(x-1)^2(x+2)^4 + 4(x-1)^3(x+2)^3.$$

Bonus: Factor following expression

$$\frac{3(x+2)^2(x-3)^2 - (x+2)^3(2)(x-3)}{(x-3)^4}.$$

1. (a) Yes (b) Yes

2. (a) Domain: $-4 \le x \le 10$ or $[-4, 10]$

Range: $-3 \le y \le 5$ or $[-3, 5]$

(b) $f(2) = 0$

(c) $x = 4$ and $x = 8$

(d) x-int: $x = 2$ and $x = 9$

y-int: $y = -1$

(e) $(-2, 6)$ or $-2 < x < 6$

(f) $(-4, -2)$ and $(6, 10)$ or $-4 < x < -2$ and $6 < x < 10$

(g) f has a local min at $x = -2$ and $f_{min} = -3$

f has a local max at $x = 6$ and $f_{max} = 5$

(i) $\frac{\Delta y}{\Delta x} = \frac{f(4) - f(0)}{4 - 0} = \frac{3 - (-1)}{4} = \frac{4}{4} = 1$

(j) $(0, f(0)) = (0, -1)$ and use point-slope form:

$y - y_1 = m(x - x_1)$ $\quad m = \frac{\Delta y}{\Delta x} = 1$

$y - (-1) = 1 \cdot (x - 0)$

$y = x - 1$

or, noticing that slope $m = 1$ and the y-int is $y = -1$,

We are able write it out directly:

$y = mx + b \Rightarrow y = x - 1$.

3. $f(x) = x^2 + 3x + 1$

$f(x+h) = (x+h)^2 + 3(x+h) + 1$

$= x^2 + 2hx + h^2 + 3x + 3h + 1$

$f(x+h) - f(x) = x^2 + 2hx + h^2 + 3x + 3h + 1 - (x^2 + 3x + 1)$

$= x^2 + 2hx + h^2 + 3x + 3h + 1 - x^2 - 3x - 1$

$= 2hx + h^2 + 3h$

$$\frac{f(x+h) - f(x)}{h} = \frac{2hx + h^2 + 3h}{h} = \frac{h(2x + h + 3)}{h}$$

$$= 2x + h + 3.$$

4. (a) C = charge for first 500 miles + charge for remaing $(x-500)$ miles.

$C = .60 \cdot 500 + .50 \cdot (x - 500)$

(b) $C = .60 \cdot 500 + .50 \cdot 700 + 2 \cdot (x - 1200)$

$(500 + 700 = 1200)$

(c) $C = .60 \cdot 500 + .50 \cdot 700 + 2 \cdot 800$

5. $y = \sqrt[3]{x} \xrightarrow{①} y = \sqrt[3]{x+3} \xrightarrow{②} y = 4\sqrt[3]{x+3} \xrightarrow{③} y = 4\sqrt[3]{x+3} - 1$

① $f(x) \to f(x+3)$: shift left 3 units.

② $f(x) \to 4f(x)$: Vertical stretch by a factor of 4.

③ $f(x) \to f(x) - 1$: Shift down 1 unit.

6. (a) : $f(x) \to f(x-2)$: $y = (x-2)^3$

(b) : $f(x) \to 4f(x)$: $y = 4(x-2)^3$

(c) : $f(x) \to -f(x)$: $y = -4(x-2)^3$

7. (a) : Revenue = price * quantity

$$R = P \cdot x \qquad (P = -2x + 40)$$

$$R = (-2x+40) \cdot x = -2x^2 + 40x$$

(b) $x = 5$: $R(5) = -2 \cdot 5^2 + 40 \cdot 5 = -50 + 200 = 150$.

(c). It's asking about the vertex of parabola. $a = -2 < 0$. opens down $\Rightarrow$ It has a local max at (h, k). ($f_{max} = k$ at $x = h$).

$$x = h = \frac{-b}{2a} = \frac{-40}{2 \cdot (-2)} = \frac{40}{-4} = 10.$$

$$k = \frac{4ac - b^2}{4a} = \frac{4 \cdot (-2) \cdot 0 - 40^2}{4 \cdot (-2)} = \frac{-1600}{-8} = 200.$$

Therefore. $x = 10$ maximizes revenue, and the max revenue is 200.

8. $3(x-1)^2(x+2)^4 + 4(x-1)^3(x+2)^3$

$= (x-1)^2 (x+2)^3 [3(x+2) + 4(x-1)]$

$= (x-1)^2 (x+2)^3 [3x + 6 + 4x - 4]$

$= (x-1)^2 (x+2)^3 (7x+2)$

Bonus : $\dfrac{3(x+2)^2(x-3)^2 - (x+2)^3(2)(x-3)}{(x-3)^4}$

$= \dfrac{(x+2)^2 (x-3) [3(x-3) - 2(x+2)]}{(x-4)^{4-3}}$

$= \dfrac{(x+2)^2 [3x - 9 - 2x - 4]}{(x-4)^3} = \dfrac{(x+2)^2 (x-13)}{(x-4)^3}$.

FORMULAS

$(a+b)(c+d) = ac + ad + bc + bd$
$(a+b)^2 = a^2 + 2ab + b^2$
$(a-b)^2 = a^2 - 2ab + b^2$
$a^2 + 2ab + b^2 = (a+b)^2$
$a^2 - 2ab + b^2 = (a-b)^2$

Quadratic formula for quadratic equation

Solutions for a quadratic equation: $ax^2 + bx + c = 0$ are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Factoring

$ax^2 + bx + c = a(x - x_1)(x - x_2)$, where x_1, x_2 are solutions of equation $ax^2 + bx + c = 0$.

Transformations

I SHIFTING

Given a function $y = f(x)$ and a constant $c > 0$,

(a) $y = f(x) + c$ shifts graph **UP** c unit (add c to y-values)

(b) $y = f(x) - c$ shifts graph **DOWN** c unit (subtract c from y-values)

(c) $y = f(x + c)$ shifts graph **LEFT** c unit (subtract c from x-values)

(d) $y = f(x - c)$ shifts graph **RIGHT** c unit (add c to x-values)

II STRETCHING/COMPRESSING

Given a function $y = f(x)$ and a constant $c > 1$,

(a) $y = c * f(x)$ **VERTICAL STRETCH** by a factor of c (multiply y's by c)

(b) $y = 1/c * f(x)$ **VERTICAL COMPRESS** by a factor of $1/c$ (divide y's by c)

(c) $y = f(c * x)$ **HORIZONTAL COMPRESS** by a factor of $1/c$ (divide x's by c)

(d) $y = f(1/c * x)$ **HORIZONTAL STRETCH** by a factor of c (multiply x's by c)

III REFLECTING

(a) $y = -f(x)$ Reflects graph about the X-**AXIS**

(b) $y = f(-x)$ Reflects graph about the Y-**AXIS**