Name_____ Test 4- MA 107-003 Spring 2009, Apr 15 RowLetter____

(Put your name on both the problem sheet and your answer sheets)

Answer following questions. Show **ALL OF YOUR WORK** to get full credit.

1. How long does it take for an investment to double in value if it is invested at 20% per annum compounded semiannually?

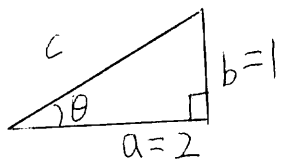
2. Suppose the half-life of some radioactive substance is 100 years. Find the decay rate of this radioactive substance. (Hint: it follows the exponential law)

3. Find the region bounded by following curves (sketch curves and shade the bounded region) and identify all vertices:

$$y = 1, \quad y - x = 1, \quad y = 5 - x.$$

4. Convert following angles between degrees and radians:
(a) 10°; (b) $-\frac{\pi}{10}$.

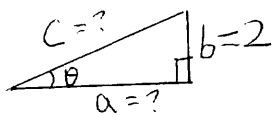
5. Find values of $\sin(\theta)$, ~~$\tan(\theta)$~~ and $\sec(\theta)$ for the following right triangle:



6. Find the exact value of following expression:

$$\sin(30°) + \cos^2(\frac{\pi}{4}).$$

7. Let $\sin(\theta) = \frac{1}{4}$ and $b = 2$.
(a) Find a and c.
(b) Find values of $\cos(\theta)$ ~~and $\csc(\theta)$~~



8. To measure the height of a building, two sightings are taken a distance of 50 feet apart. If the first angle of elevation is 40° and the second is 32°, what is the height of the building?

Bonus: Let (-3,4) be a point on the terminal side of some angle θ. Here θ is some general angel and actually not an acute one. Find all six trig functions of this angle θ.

$r = \sqrt{(-3)^2+4^2} = \sqrt{25} = 5$

hyp — r
adj — x-comp
op — y-comp

$\sin\theta = \frac{y\text{-comp}}{r} = \frac{4}{5}$

$\cos\theta = \frac{x\text{-comp}}{r} = \frac{-3}{5} = -\frac{3}{5}$

$\tan\theta = \frac{y\text{-comp}}{x\text{-comp}} = \frac{4}{-3} = -\frac{4}{3}$

$\cot\theta = \frac{x\text{-comp}}{y\text{-comp}} = \frac{-3}{4} = -\frac{3}{4}$

$\sec\theta = \frac{r}{x\text{-comp}} = \frac{5}{-3} = -\frac{5}{3}$

$\csc\theta = \frac{r}{y\text{-comp}} = \frac{5}{4}$

1. $A = A_0(1+\frac{r}{n})^{nt}$

$A = 2A_0$, $r = 20\% = .2$, $n = 2$

$\frac{2A_0}{A_0} = \frac{A_0}{A_0}(1+\frac{.2}{2})^{2t}$

$2 = (1+0.1)^{2t}$

$2 = 1.1^{2t} \Leftrightarrow \ln 2 = 2t \ln 1.1 \Rightarrow t = \frac{\ln 2}{2\ln 1.1}$

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2. $A = A_0 e^{kt}$.

$A = \frac{1}{2}A_0 \Rightarrow t = 100$.

$\frac{1}{2}A_0 = A_0 e^{k100}$

$\frac{1}{2} = e^{100k}$

$\ln\frac{1}{2} = 100k \Rightarrow k = \frac{\ln\frac{1}{2}}{100}$

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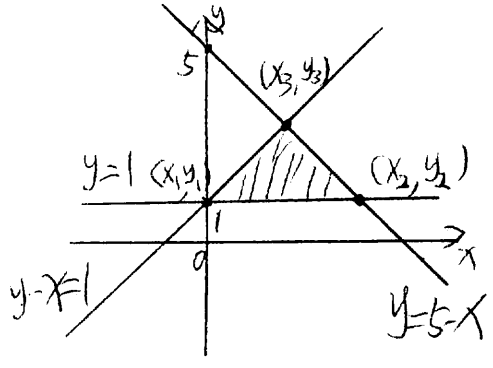
3. Sketch the curves & shade the bounded region



$y = 1$

$y - x = 1 \Leftrightarrow y = x+1$

$y = 5-x$

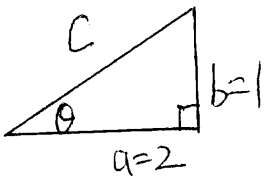
$(x_1, y_1)\begin{pmatrix} y=1 \\ y=x+1 \end{pmatrix}$ $\begin{cases} y=1 \\ y=x+1 \end{cases} \Rightarrow \begin{cases} y=1 \\ x=-1 \end{cases} \Rightarrow (-1, 1)$

$(x_2, y_2)\begin{pmatrix} y=1 \\ y=5-x \end{pmatrix}$ $\begin{cases} y=1 \\ y=5-x \end{cases} \Rightarrow \begin{cases} y=1 \\ x=4 \end{cases} \Rightarrow (4, 1)$

$(x_3, y_3)\begin{pmatrix} y=x+1 \\ y=5-x \end{pmatrix}$ $\begin{cases} y=x+1 \\ y=5-x \end{cases} \Rightarrow \begin{cases} x+1=5-x \\ x+x=5-1 \\ 2x=4 \\ x=2 \Rightarrow y=3 \end{cases} \Rightarrow (2, 3)$

4. (a) $10^\circ = 10 \cdot 1^\circ = 10 \cdot \frac{\pi}{180} = \frac{\pi}{18}$ radian

(b) $-\frac{\pi}{10} = -\frac{\pi}{10} \cdot 1 \text{ radian} = -\frac{\pi}{10} \cdot \frac{180^\circ}{\pi} = -18^\circ$

5. 

$c = \sqrt{a^2+b^2} = \sqrt{1^2+2^2} = \sqrt{5}$

$\sin\theta = \frac{b}{c} = \frac{1}{\sqrt{5}} = \frac{1\cdot\sqrt{5}}{\sqrt{5}\cdot\sqrt{5}} = \frac{\sqrt{5}}{5}$

$\tan\theta = \frac{b}{a} = \frac{1}{2}$

$\sec\theta = \frac{c}{a} = \frac{\sqrt{5}}{2}$

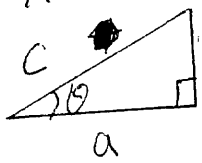
6. $\sin(30^\circ) + \cos^2(\frac{\pi}{4})$

$= \frac{1}{2} + \cos^2(45^\circ)$

$= \frac{1}{2} + (\frac{\sqrt{2}}{2})^2$

$= \frac{1}{2} + \frac{1}{2} = 1$

7. $\sin\theta = \frac{1}{4}$, $b = 2$.

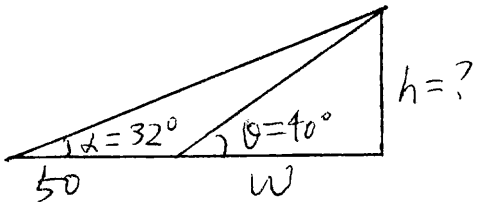


$\sin\theta = \frac{b}{c} = \frac{2}{c} = \frac{1}{4}$ $\quad 2\cdot 4 = 1\cdot c$, $c = 8$

$a = \sqrt{c^2-b^2} = \sqrt{8^2-2^2} = \sqrt{64-4} = \sqrt{60}$

$\cos\theta = \frac{a}{c} = \frac{\sqrt{60}}{8}$

$\csc\theta = \frac{c}{b} = \frac{8}{2} = 4$

8. 

We use tangent function here.

$$\begin{cases} \tan 40^\circ = \frac{h}{w} \\ \tan 32^\circ = \frac{h}{50+w} \end{cases}$$

$$\Rightarrow \begin{cases} h = w\tan 40^\circ \\ h = (50+w)\tan 32^\circ = 50\cdot\tan 32^\circ + w\tan 32^\circ \end{cases}$$

$\Rightarrow w\tan 40^\circ = 50\cdot\tan 32^\circ + w\tan 32^\circ$

$w\tan 40^\circ - w\tan 32^\circ = 50\cdot\tan 32^\circ$

$w(\tan 40^\circ - \tan 32^\circ) = 50\cdot\tan 32^\circ$

$$w = \frac{50\cdot\tan 32^\circ}{\tan 40^\circ - \tan 32^\circ}$$

$$h = w\cdot\tan 40^\circ = \frac{50\cdot\tan 32^\circ\cdot\tan 40^\circ}{\tan 40^\circ - \tan 32^\circ}$$