Name____________________ **Final Exam- MA 107-003 Spring 2009, Apr 29: 1-4PM**

(Put your name on both the problem sheet and your answer sheets)

Answer following questions. Show **ALL OF YOUR WORK** to get full credit.

1. Find the vertex of the graph of the quadratic function $f(x) = 2x^2 - 4x + 3$.

2. List all possible rational zeroes of $f(x) = x^3 - 2x^2 - 3$. Find the quotient and remainder if $f(x)$ is divided by $(x-1)$, ie, if we write $f(x) = q(x) \cdot (x-1) + r(x)$, find $q(x)$ and $r(x)$ in the expression.

3. Factor completely: $x^4(x-8)^3 + x^5(x-8)^2$.

4. If $f(x) = 2x - x^2$, find the difference quotient of $f(x)$: $\frac{f(x+h)-f(x)}{h}$.

5. Find the inverse function $g^{-1}(x)$ of the one-to-one function $g(x) = x^2 - 3, x \geq 0$.
(Note $g^{-1}(x) \neq \frac{1}{g(x)}$!)

6. Given the polynomial $f(x) = x^3(x-2)^2(x-4)$. Find the degree, leading coefficient, end behavior. all zeros and their multiplicity, and touches/crosses the x-axis. Then use this information to sketch the graph of the function.

7. For the rational function $R(x) = \frac{x^2+5x+6}{x^2+4x+4}$ find the vertical and horizontal asymptotes and the x-intercept(s) of the function.

8. If \$21,000 is invested at an interest rate of 8% per annum compounded quarterly, find the value in the account after 10 years.

9. Write $\log \frac{a^3}{b^6\sqrt{d}}$ in expanded form.

10. Solve for x:
   a. $\log(3-x) = 2$;
   b. $\log_6(x-1) + \log_6(x) = 1$;
   c. $3^{x-2} = 7^x$.

11. Use special triangles to find exact values of $\sin^2(45°) + \cot(30°)$.

12. Find the exact value of each of the remaining trig functions if $\cos(\theta) = -\frac{4}{5}$ and θ is in Quadrant II.

13. Find the reference angles for: a. $\theta = 550°$; b. $\theta = -130°$.

14. Solve the inequality $f(x) = \frac{x+1}{x-3} < 0$.

Bonus. Let $f(x)$ be a complex polynomial of degree 5 with real coefficients. We know it has three zeros: $1, 3i, 2+i$. Find the rest of its zeros.

15. A wire 90 feet long is attached to the top of a radio transmission tower, making an angle of 25° with the ground. How high is the tower?

Formulas you might use

Quadratic formula for quadratic equation:

$$ax^2 + bx + c = 0 \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \text{ (if } b^2 - 4ac \geq 0)$$

Log Identities: (x, a, b, M **and** N **are all positive numbers.** y **and** r **are real numbers)**

$$y = \log_a(x) \Leftrightarrow a^y = x$$
$$\log_a(MN) = \log_a(M) + \log_a(N)$$
$$\log_a(\frac{M}{N}) = \log_a(M) - \log_a(N)$$
$$\log_a(M^r) = r \cdot \log_a(M)$$
$$\log_a(M) = \frac{\log_b(M)}{\log_b(a)}$$

Trig Identities

$$\tan(\theta) \cdot \cot(\theta) = 1 \; (\tan(\theta) = \frac{1}{\cot(\theta)}, \; \cot(\theta) = \frac{1}{\tan(\theta)})$$
$$\sec(\theta) \cdot \cos(\theta) = 1 \; (\sec(\theta) = \frac{1}{\cos(\theta)}, \; \cos(\theta) = \frac{1}{\sec(\theta)})$$
$$\csc(\theta) \cdot \sin(\theta) = 1 \; (\csc(\theta) = \frac{1}{\sin(\theta)}, \; \sin(\theta) = \frac{1}{\csc(\theta)})$$
$$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}; \; \cot(\theta) = \frac{\cos(\theta)}{\sin(\theta)}$$
$$\sin^2(\theta) + \cos^2(\theta) = 1$$
$$\tan^2(\theta) + 1 = \sec^2(\theta)$$
$$\cot^2(\theta) + 1 = \csc^2(\theta)$$

key to final exam

1. vertex $\left(-\frac{b}{2a}, \frac{4ac-b^2}{4a}\right)$. $a=2$, $b=-4$, $c=3$

$$\frac{-b}{2a}=\frac{-(-4)}{2\cdot 2}=\frac{4}{4}=1$$

$$\frac{4ac-b^2}{4a}=\frac{4\cdot 2\cdot 3-(-4)^2}{4\cdot 2}=\frac{24-16}{8}=\frac{8}{8}=1.$$

vertex : $\boxed{(1,1)}$

2. $a_0=-3 \Rightarrow p=\pm 1, \pm 3,$

$a_0=1 \Rightarrow q=\pm 1$

$\Rightarrow \boxed{\frac{p}{q}=\pm 1, \pm 3} \Leftarrow$ all possible rational zeroes.

$$\begin{array}{r} x^2 - x - 1 \quad \Leftarrow q(x) \\ x-1 \enclose{longdiv}{x^3-2x^2+0x-3} \\ -(x^3-x^2) \\ \hline -x^2+0x \\ -(-x^2+x) \\ \hline -x-3 \\ -x+1 \\ \hline -4 \Leftarrow r(x) \end{array}$$

so $\boxed{\begin{array}{l} q(x)=x^2-x-1 \\ r(x)=-4. \end{array}}$

3. $x^4(x-8)^3+x^5(x-8)^2$

$=x^4(x-8)^2[(x-8)+x]$

$=x^4(x-8)^2(2x-8)$

$=x^4(x-8)^2\cdot 2\cdot(x-4)$

$=\boxed{2x^4(x-8)^2\cdot(x-4)}$

4. $f(x) = 2x - x^2$

$f(x+h) = 2(x+h) - (x+h)^2$

$= 2x + 2h - (x^2 + 2hx + h^2)$

$= 2x + 2h - x^2 - 2hx - h^2$

$f(x+h) - f(x) = 2x + 2h - x^2 - 2hx - h^2 - (2x - x^2)$

$= \cancel{2x} + 2h - \cancel{x^2} - 2hx - h^2 - \cancel{2x} + \cancel{x^2}$

$= 2h - 2hx - h^2$

$$\frac{f(x+h) - f(x)}{h} = \frac{2h - 2hx - h^2}{h} = \frac{\cancel{h}(2 - 2x - h)}{\cancel{h}} = \boxed{2 - 2x - h}$$

5. $g(x) = x^2 - 3, \ x \geq 0$

$y = x^2 - 3, \ x \geq 0$

$x = y^2 - 3$

$x + 3 = y^2$

$y = \sqrt{x+3}$ or $y = -\sqrt{x+3}$

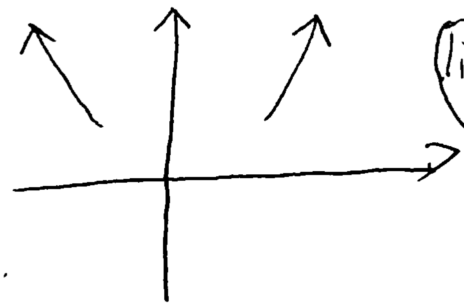
Domain of g = Range of g^{-1}

$x \geq 0 \iff y \geq 0$

$\Rightarrow \boxed{y = \sqrt{x+3} \qquad x \geq -3}$

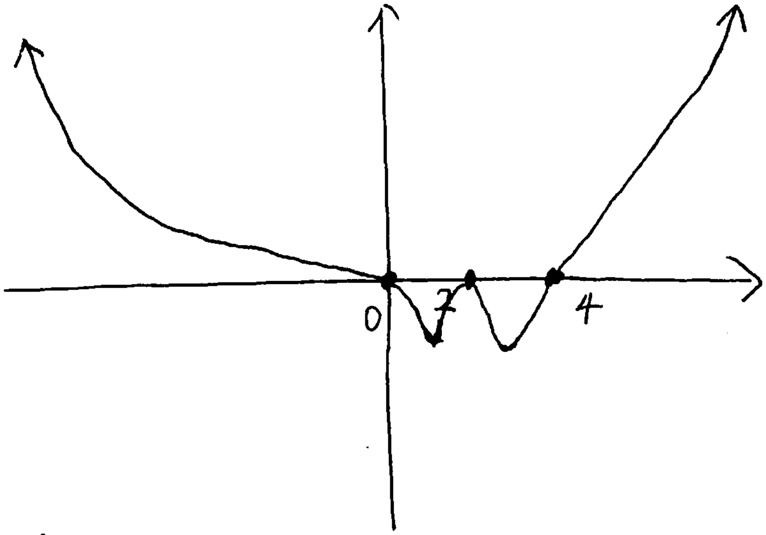
6. degree : $n = 3 + 2 + 1 = \boxed{6}$ Leading Coefficient $\boxed{a_6 = 1^3 1^2 1 = 1}$

endbehavior : $a_n x^n = 1 \cdot x^6 = x^6 \Rightarrow$



like x^2 b/c $n=6$ is even#

zero	mult	even/odd	touches/crosses
$x=0$	3	odd	crosses
$x=2$	2	even	touches
$x=4$	1	odd	crosses



7. $R(x) = \frac{x^2+5x+6}{x^2+4x+4} = \frac{(x+2)(x+3)}{(x+2)^2} = \frac{p(x)}{q(x)}$

If $x+2 \neq 0$, ie $x \neq -2$, we also have:

$R(x) = \frac{\cancel{(x+2)}(x+3)}{\cancel{(x+2)}(x+2)} = \frac{x+3}{x+2}$ ⇐ in lowest terms.

VA: $x+2=0 \Rightarrow \boxed{x=-2}$

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x-int: $p(x)=0$ but $q(x) \neq 0$ ⇐ denomenator $\neq 0$.

$p(x) = (x+2)(x+3) = 0 \Rightarrow x=-2$ or $x=-3$

$q(x) \neq 0 \Rightarrow x \neq -2$

So x-int: $x=-3$.

HA: $y = \frac{\text{Leading coefficient of } p(x)}{\text{Leading coefficient of } q(x)}$ if degree(p) = degree(q)

$y = 0$ if degree(p) < degree(q)

Does not exist if ---- > ----

Here degree (P) = 2 = degree (q)

$\Rightarrow y = \frac{\text{Leading coefficient of P}}{\text{Leading coefficient of q}} = \frac{1}{1} = 1$

$\Rightarrow \boxed{y=1}$

8. Formula for compound interest:

$A=P(1+\frac{r}{n})^{nt}$ compounded n times a year

$A=Pe^{rt}$ Compounded continuously.

Here we use the first formula: $n=4$.

Also. $P=21{,}000$, $r=8\%=.08$, $t=10$.

$$A=P(1+\frac{r}{n})^{nt}$$
$$=21{,}000\cdot(1+\frac{.08}{4})^{4\cdot 10}$$
$$=\boxed{21{,}000\cdot 1.02^{40}}$$

9. $$\log\frac{a^3}{b^5\cdot\sqrt{d}}=\log(a^3)-\log(b^5\sqrt{d})$$
$$=3\log a-[\log b^5+\log\sqrt{d}]$$
$$=3\log a-\log b^5-\log\sqrt{d}$$
$$=3\log a-5\log b-\log d^{\frac{1}{2}}$$
$$=\boxed{3\log a-5\log b-\tfrac{1}{2}\log d}$$

10. (a) $\log(3-x)=2$ $\Leftarrow$ common log : base $a=10$.

$y=\log_a x \Leftrightarrow a^y=x$

$$\Rightarrow 10^2=3-x \Rightarrow 10^2-3=-x$$
$$97=-x$$
$$x=-97.$$

Check: $3-x=3-(-97)=100>0$ ✓.

$\Rightarrow \boxed{x=-97}$ is the solution

(b) $\log_6(x-1) + \log_6(x) = 1$

$\log_6[(x-1)\cdot x] = 1$

$6^1 = (x-1)\cdot x$

$6 = x^2 - x$

$x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$

$\Rightarrow x_1 = 3 \quad x_2 = -2$.

check: $x_1 = 3$: $x-1 = 3-1 = 2 > 0$. $x = 3 > 0$ ✓

$x_2 = -2$: $x-1 = -2-1 = -3 < 0$ ✗

$\Rightarrow \boxed{x = 3}$ is the only solution

(c) $3^{x-2} = 7^x$

$\ln 3^{x-2} = \ln 7^x$

$(x-2)\ln 3 = x \cdot \ln 7$

$x \cdot \ln 3 - 2\ln 3 = x \cdot \ln 7$

$x \cdot \ln 3 - x \cdot \ln 7 = 2 \cdot \ln 3$

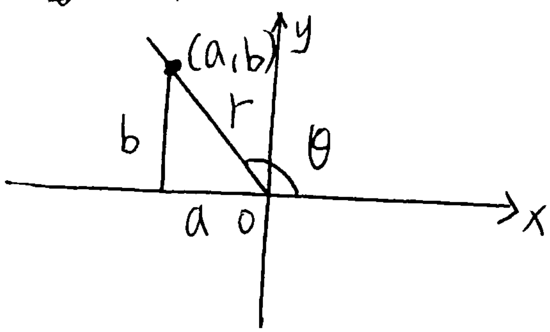
$x(\ln 3 - \ln 7) = 2\ln 3$

$$\boxed{x = \frac{2 \cdot \ln 3}{\ln 3 - \ln 7}}$$

11. $\sin(45^\circ) = \frac{\sqrt{2}}{2}$. $\cot(30^\circ) = \sqrt{3}$.

$$\Rightarrow \sin^2(45^\circ) + \cot(30^\circ) = \left(\frac{\sqrt{2}}{2}\right)^2 + \sqrt{3}$$
$$= \frac{2}{4} + \sqrt{3}$$
$$= \boxed{\frac{1}{2} + \sqrt{3}}$$

12. $\cos(\theta) = -\frac{4}{5}$. $\theta \in QII$.



$\theta \in QII \Rightarrow a<0,\ b>0.\ r>0$

$\cos\theta = \frac{a}{r} = -\frac{4}{5} = \frac{-4}{5}$.

Let $a=-4$ $r=5$.

$$\Rightarrow b = \pm\sqrt{5^2-(-4)^2}$$
$$= \pm 3$$

$b>0 \Rightarrow b=3$.

$\sin\theta = \frac{b}{r} = \frac{3}{5}$

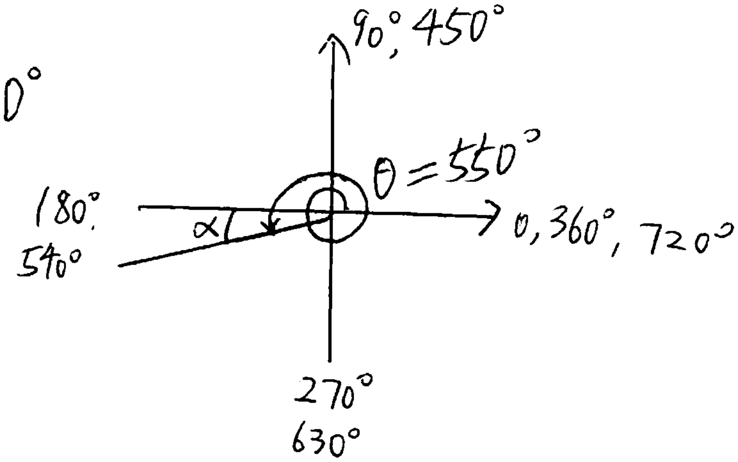
$\tan\theta = \frac{b}{a} = \frac{3}{-4} = -\frac{3}{4}$

$\cot\theta = \frac{a}{b} = \frac{-4}{3} = -\frac{4}{3}$

$\sec\theta = \frac{r}{a} = \frac{5}{-4} = -\frac{5}{4}$

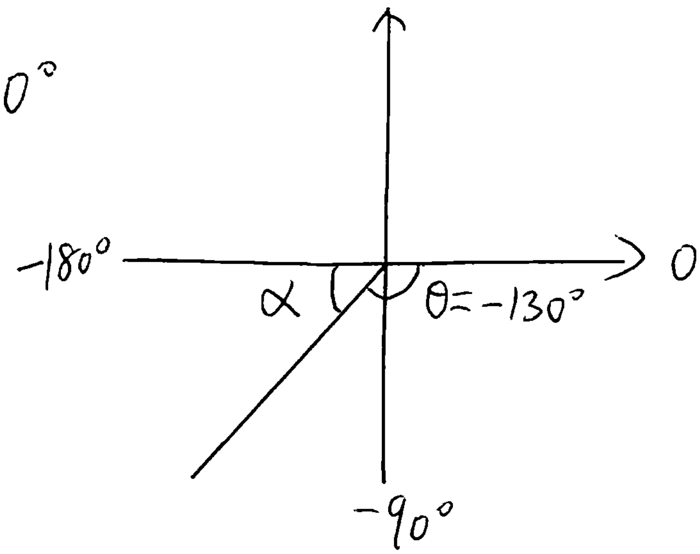
$\csc\theta = \frac{r}{b} = \frac{5}{3}$.

13. a: $\theta = 550°$



$$540° + \alpha = 550° \Rightarrow \boxed{\alpha = 10°}$$

b: $\theta = -130°$



$$-180° + \alpha = -130°$$

$$\alpha = -130° - (-180°)$$

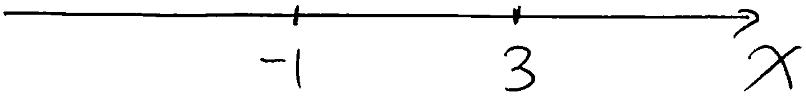
$$= -130° + 180°$$

$$= \boxed{50°}$$

14. $f(x) = \frac{x+1}{x-3} < 0.$

zero(s) of f: $x+1 = 0 \Rightarrow x = -1$

points where f is not defined: $x-3=0 \Rightarrow x=3$
(denomenator $=0$)



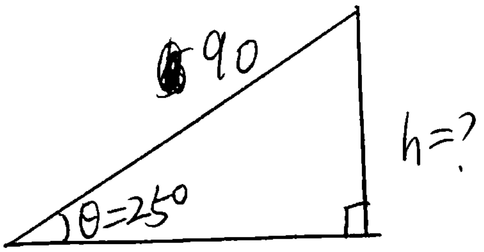
$-2 \in (-\infty, -1)$ $+$ $-$, $f(-2) = \frac{-2+1}{-2-3} = \frac{-1}{-5} = \frac{1}{5} > 0$

$0 \in (-1, 3)$ $+$ $-$ $f(0) = \frac{0+1}{0-3} = -\frac{1}{3} < 0$

$4 \in (3, \infty)$ $+$ $-$ $f(4) = \frac{4+1}{4-1} = \frac{5}{1} = 5 > 0$

$f(x) < 0 \Leftrightarrow$ $\boxed{x \in (-1, 3) \text{ or } -1 < x < 3}$

15.



$r = 90$, $h = ?$ Hyp & opp $\Rightarrow$ use sine function.

$\sin\theta = \frac{opp}{hyp}$ $\Rightarrow$ $\sin 25° = \frac{h}{90}$ $\Rightarrow$ $\boxed{h = 90 \cdot \sin 25°}$

Bonus: $1, 3i, 2+i, \overline{3i}, \overline{2+i}$

$\boxed{-3i, \quad 2-i}$