MA107- Exponentials and Logarithms Worksheet

1. Find to two decimal places: $log_5(11)$

2. Graph $y = log(x-3)$. Find it's domain and range.

3. Find the exact value (no decimal answers - Use laws of logs to find without a calculator) of

$$log_4 20 + 3log_4 2 - log_4 10$$

4. Solve for x: $log_3(x-1) - log_3(x+4) = 2$ (hint: condense and convert)

5. Solve for x: $3^{x+1} = 6^x$ (hint: take the ln of both sides)

6. Solve for x: $log_5(x+4) + log_5(x) = 1$ (hint: condense and convert)

7. Solve for t: $-6e^{0.4t} + 10 = 0$ (hint: isolate the exponential term first)

8. Psychologists sometimes use the function $L = A(1 - e^{-kt})$ to measure the amount L learned after time t minutes. The number A represents the amount to be learned, and the number k measures the rate of learning. Suppose that a student has an amount A of 200 vocabulary words to learn. A psychologist determines that the student learned 20 vocabulary words after 5 minutes.
   a. Determine the rate of learning k.
   b. Approximately how many words will the student have learned after 15 minutes?

9. At a nearby high school, someone overhears the principal say that school will be closed a day early this week. The number of people, N, who hear this rumor in t minutes is given by the formula

$$N = N_f - N_f e^{-.15t}$$

where N_f is the fixed population of the school. If the school has 1800 students and staff members, how many minutes will it take for 3/4 of the school to hear the rumor?

10. The energy E (in ergs) released during an earthquake of magnitude R (from the Richter scale) may be approximated by the formula $log(E) = 11.4 + (1.5)R$. Find the energy that was released during one of the biggest earthquakes in history which took place in 1933 in Japan and had a magnitude of 8.9.

1. $\log_5 11 = \frac{\ln 11}{\ln 5} \approx \frac{2.39785}{1.60944} \approx 1.4899$

2. Domain: $x-3>0 \Leftrightarrow x>3$. Range: $\mathbb{R}$.

3. $\log_4 20 + 3\log_4 2 - \log_4 10 = \log_4 20 + \log_4 2^3 - \log_4 10$

$$= \log_4 (20 \cdot 2^3) - \log_4 10$$

$$= \log_4 \left(\frac{20 \cdot 2^3}{10}\right)$$

$$= \log_4 (16) = 2$$

4. $\log_3(x-1) - \log_3(x+4) = 2$

$$\log_3 \frac{x-1}{x+4} = 2 \Leftrightarrow 3^2 = \frac{x-1}{x+4}$$

$$9 = \frac{x-1}{x+4}$$

$$9(x+4) = x-1$$

$$9x+36 = x-1$$

$$8x = -37$$

$$x = \frac{-37}{8}$$

check: $x-1 = \frac{-37}{8} - 1 < 0 \Rightarrow x = -\frac{37}{8}$ is not a solution

$\Rightarrow$ No solution

5. $3^{x+1} = 6^x \Leftrightarrow \ln 3^{x+1} = \ln 6^x$

$$\Leftrightarrow (x+1)\ln 3 = x \cdot \ln 6$$

$$x \cdot \ln 3 + \ln 3 = x \cdot \ln 6$$

$$\ln 3 = x \cdot \ln 6 - x \cdot \ln 3$$

$$\ln 3 = x \cdot (\ln 6 - \ln 3)$$

$$x = \frac{\ln 3}{\ln 6 - \ln 3}$$

6. $\log_5(x+4)+\log_5(x)=1$

$\log_5[(x+4)x]=1 \Leftrightarrow (x+4)x=5^1=5$

$x^2+4x=5$

$x^2+4x-5=0$

$(x+5)(x-1)=0$

$x=-5$ or $x=1$.

Check: $x=-5$: $x+4=-5+4=-1<0$ ✗

$x=1$. $x+4=1+4=5>0$ ✓

$x=1>0$ ✓

Solution: $x=1$.

7. $-6e^{.4t}+10=0 \Rightarrow 10=6e^{.4t}$

$\Rightarrow e^{.4t}=\frac{10}{6}=\frac{5}{3}$

$.4t=\ln\frac{5}{3}$

$t=\frac{\ln\frac{5}{3}}{.4}$

8. (a) $A=200$. $L=20$ when $t=5$.

$\Rightarrow 20=200(1-e^{-k5})$

$.1=\frac{20}{200}=1-e^{-5k}$

$e^{-5k}=1-.1$

$e^{-5k}=.9$

$-5k=\ln .9$

$k=\frac{\ln .9}{-5}=.021$

(b) $t=15$. $L=?$

$L=200(1-e^{-.021\cdot 15})$

$=200(1-e^{-.315})$

$=200(1-.7298)$

$=54$

9. $N_f = 1800.$

$N = \frac{3}{4}N_f \quad , \quad t = ?$

$$\frac{3}{4}N_f = N_f - N_f e^{-.15t}$$

$$\frac{3}{4}N_f - N_f = -N_f e^{-.15t}$$

$$\frac{-\frac{1}{4}N_f}{-N_f} = \frac{-N_f e^{-.15t}}{-N_f}$$

$$\frac{1}{4} = e^{-.15t}$$

$$-.15t = \ln\frac{1}{4}$$

$$t = \frac{\ln\frac{1}{4}}{-.15} = 9.24.$$

10. $R = 8.9 \quad , \quad E = ?$

$$\log E = 11.4 + 1.5 * 8.9$$

$$\log E = 11.4 + 13.35$$

$$\log E = 24.75$$

(base is 10 : common log)

$$\Leftrightarrow \quad E = 10^{24.75}$$

$$\cancel{= 5.62 \times 10^{24}}$$

$$= 10^{24 + .75}$$

$$= 10^{24} \cdot 10^{.75}$$

$$= 10^{24} \cdot 5.62 = 5.62 * 10^{24}$$