

NORTH CAROLINA STATE UNIVERSITY  
 Department of Mathematics  
 MA 107 - Calculus - Related Factoring

In calculus, you will need to factor and simplify expressions resulting from finding derivatives of functions using the product or quotient rules.

To better prepare you for success in calculus MA 107 offers drill in this type of factoring. All these problems use the distributive property for factoring:  $ab + ac = a(b + c)$

Ex: Factor completely to simplify

$$\begin{aligned} & (2x+1)(6x) + (3x^2 - 2)(2) \\ &= 2[(2x+1)(3x) + (3x^2 - 2)] \\ &= 2[6x^2 + 3x + 3x^2 - 2] \\ &= 2[9x^2 + 3x - 2] \\ &= 2(3x+2)(3x-1) \end{aligned}$$

Ex: Factor completely to simplify:

$$\begin{aligned} & 3(x-1)^2(x+2)^4 + 4(x-1)^3(x+2)^3 \\ &= (x-1)^2(x+2)^3[3(x+2) + 4(x-1)] \\ &= (x-1)^2(x+2)^3[3x+6+4x-4] \\ &= (x-1)^2(x+2)^3(7x+2) \end{aligned}$$

Exercises. Factor completely and simplify.

1.  $2(x-1)^2 + 5(x-1)$

2.  $9x^2(x^4 + 2) + 3x^3(4x^3)$

3.  $3x^2(4x-12)^2 + x^3(2)(4x-12)(4)$

4.  $5(x^2 + 4)^4(2x)(x-2)^4 + (x^2 + 4)^5(4)(x-2)^3$

5.  $3(x-1)^2(x+2)^4 + 4(x-1)^3(x+2)^3$

6.  $2(x+1)(x-1)^{-2} - 2(x+1)^2(x-1)^{-3}$

7.  $6(x^2 + 3x + 1)(2x + 3)^2 + 6(x^2 + 3x + 1)^2$

8.  $60x^3(-3x^2)^5(5x^4 - 1)^3 - 30x(5x^4 - 1)^3(-3x^2)^5$

9.  $\frac{2x(x+6)^4 - x^2(4)(x+6)^3}{(x+6)^8}$

10.  $\frac{(x-1)^3 - 3(x-5)(x-1)^2}{(x-1)^6}$

11.  $\frac{(x-1)^2(1-2x) - 2(2+x-x^2)(x-1)}{(x-1)^4}$

12.  $\frac{8(1-3x)^2(2x-1)^3 + 6(2x-1)^4(1-3x)}{(1-3x)^4}$

13.  $\frac{3(x+2)^2(x-3)^2 - (x+2)^3(2)(x-3)}{(x-3)^4}$

14.  $\frac{3(x+2)^2 - 6x(x+2)}{x+2}$

MA-107 - Calculus - Related Factoring wkst - SOLUTIONS

$$1) (x-1)(2(x-1)+5) = (x-1)(2x+3)$$

$$2) 3x^2 (3(x^4+2)+x(4x^3)) = 3x^2 (7x^4+6)$$

$$3) x^2 (4x-12) (3(4x-12) + x(2)(4)) = x^2 (4x-12)(20x-36)$$

$$4) 2(x^2+4)^4 (x-2)^3 (5(x)(x-2) + (x^2+4)(2))$$

$$= 2(x^2+4)^4 (x-2)^3 (7x^2 - 10x + 8)$$

$$5) (x-1)^2 (x+2)^3 (3(x+2) + 4(x-1)) = (x-1)^2 (x+2)^3 (7x+2)$$

~~$$6) 2(x+1)(x-1)^{-3} (1 - (x+1)(x-1)^{-1}) = \frac{2(x+1)}{(x-1)^2} \left( \frac{(x-1) - (x+1)}{(x-1)} \right)$$

$$= \frac{2(x+1)(-2)}{(x-1)^3}$$~~

see next page.

$$7) 6(x^2+3x+1) ((2x+3)^2 + (x^2+3x+1))$$

$$= 6(x^2+3x+1)(5x^2+15x+10)$$

$$8) 30x(1-3x^2)^4 (5x^4-1)^2 (2x^2(1-3x^2) - (5x^4-1))$$

$$= 30x(1-3x^2)^4 (5x^4-1)^2 (-11x^4+2x^2+1)$$

$$9.) \frac{2x \cancel{(x+6)}^3 ((x+6) - 2x)}{(x+6)^{\cancel{3}5}} = \frac{2x(-x+6)}{(x+6)^5}$$

$$10.) \frac{\cancel{(x-1)}^4 ((x-1) - 3(x-5))}{(x-1)^{\cancel{4}4}} = \frac{-2x+14}{(x-1)^4} = \frac{-2(x-7)}{(x-1)^4}$$

$$11.) \frac{\cancel{(x-1)}^3 ((x-1)(1-2x) - 2(2+x-x^2))}{(x-1)^{\cancel{3}3}} = \frac{-\cancel{2x^2} + 3x - 1 - 4 - 2x + \cancel{2x^2}}{(x-1)^3} = \frac{x-5}{(x-1)^3}$$

$$12.) \frac{2 \cancel{(1-3x)}^3 (2x-1)^3 (4(1-3x) + 3(2x-1))}{(1-3x)^{\cancel{3}3}} = \frac{2(2x-1)^3 (1-6x)}{(1-3x)^3}$$

$$13.) \frac{(x+2)^2 \cancel{(x-3)}^3 (3(x-3) - (x+2)(2))}{(x-3)^{\cancel{3}3}} = \frac{(x+2)^2 (x-13)}{(x-3)^3}$$

$$14.) \frac{3 \cancel{(x+2)}^2 (x+2) - 2x}{\cancel{(x+2)}} = 3(-x+2) = -3x+6$$

$$\begin{aligned} \#6. & 2(x+1)(x-1)^{-2} - 2(x+1)^2(x-1)^{-3} \\ &= 2(x+1)(x-1)^{-3} [(x-1) - (x+1)] \\ &= 2 \cdot (x+1)(x-1)^{-3} \cdot [x-1-x-1] \\ &= 2(x+1)(x-1)^{-3} (-2) \\ &= -4(x+1)(x-1)^{-3} \quad \text{or} \quad \frac{-4(x+1)}{(x-1)^3} \end{aligned}$$