

Final Exam Review

1. Let $A = \begin{bmatrix} 1 & -1 & 0 & 3 \\ 3 & -2 & 1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 9 & 3 \\ -4 & 0 \\ 0 & -2 \end{bmatrix}$
- Find AB (if it exists)
 - Find BA (if it exists)
 - Find $BA+4I$, where I is the 4×4 identity matrix (if it exists).
2. Find the inverse of the matrix if it exists

$$A = \begin{bmatrix} 3 & -1 & 1 \\ 2 & 1 & 0 \\ 1 & 2 & -1 \end{bmatrix}$$

3. Solve the system of equations using matrix row operations.
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|---------------------|--------------------|
| (a) $5x + 10y = 20$ | (b) $x + 3y = 11$ |
| $x + 4y = 6$ | $3x + 6y + z = 27$ |
| | $2y + 8z = 4$ |

4. Section 2.2 problems 5,7,9

5. Determine if the following linear programming problems are standard or non-standard. If they are non-standard, be able to explain why.

Maximize $P = 8x - y + 5z$ subject to

$$2x + 5y \leq 1$$

$$y - 9z \geq 5$$

$$x + 3y - 2z \leq 6$$

$$x \geq 0, y \geq 0, z \geq 0$$

6. In the following problem, define your variables and set up the profit function and constraints and do all the necessary to arrive at your initial simplex tableau. Once you have your tableau set up with the columns appropriately labeled, circle your first pivot element.

Mary bakes cakes and sells them at the local Farmer's Market. She makes carrot cakes, chocolate cakes and cheese cakes. She makes a profit of \$12 on each carrot cake sold, \$10 on each chocolate cake sold, and \$15 on each cheese cakes sold. Each cake requires preparation time and baking time. Carrot cakes requires 0.5 hour of preparation, 0.75 hour for baking. Chocolate cakes require 0.25 hour of preparation, and 0.75 hour for baking. Cheese cakes require 0.5 hour of preparation and 1 hour for baking. In a given week, Mary has 20 hours of preparation time and 25 hours of baking time. Each week she must make at least 15 carrot cakes and 17 cheesecakes. How many of each type of cakes should Mary make to maximize her profit?

7. Solve the following linear programming problem using both the geometric and simplex method:

Maximize $C = 2x - 5y$ subject to

$$x - 6y \leq 15$$

$$x - 3 \leq y$$

$$x \geq 0, y \geq 0$$

8. Let $A = \{\text{Sally, Tom, Bob, Judy, Ralph}\}$, $B = \{\text{Sally, Judy}\}$, and $C = \{\text{Sally, Tom, Judy}\}$. Indicate whether the following statements are true or false:

a. $A \subset B$ b. $B \subset C$ c. $\{\text{Sally}\} \in A \cap B \cap C$

d. $\text{Sally} \subset A \cap B \cap C$ e. $\emptyset \in A \cap B \cap C$

9. Suppose you are asked to fill out a survey. The survey has 10 questions, and each question has 5 options (strongly agree, agree, neutral, disagree, strongly disagree). Assume that you give an answer for each question, how many different ways could you complete the survey?
10. How many different ways can you arrange the letters in REBECCA?
11. The probability that Ann will work at Gap during Christmas break is 0.6, and the probability that she will work at Best Buy is 0.5. The probability that she will do both is 0.25.
- What is the probability that Ann will work at Gap and not at Best Buy?
 - What is the probability that Ann will not work at either place?
12. A political party has decided to hold political rallies in two of the following states: Missouri, Montana, Mississippi, Michigan, Alabama, Connecticut and Pennsylvania. Assume that the states have equal probabilities of being chosen.
- What is the probability that both of the states chosen begin with the letter "M"?
 - What is the probability that both start with "M" given that at least one does?
 - What is the probability that both start with "M" given that one of the states is Michigan?
13. A box of crayons has 4 colors in it, red, blue, green and yellow...
- If you pull out 2 crayons randomly at same time, what is the sample space of that experiment?
 - List the event of one of the 2 crayons pulled being red.
 - Calculate the probability that above event occurs.
14. Given events A,B and C and their probabilities $P(A) = 0.3$, $P(B^c) = 0.45$, and $P(C) = 0.9$. $P(B \cap C) = 0.2$, $P(A \cap B) = 0.165$.
- Find $P(B|C)$.
 - Are the events B and C independent?
 - Are the events A and B independent?
 - What is $P(A \cup B)$?
15. If A and B are independent events. Suppose $P(A) = 0.4$, $P(B^c) = 0.6$.
- Find $P(A \cap B)$.
 - Find $P(A|B)$
 - Draw a Venn diagram for these two events.
16. 550 people were asked whether they own a sedan, truck or van, the results were
- | | | |
|-----------------|----------------------------|------------------|
| 235 own a sedan | 85 own a sedan and a truck | |
| 280 own a truck | 70 own a sedan and a van | |
| 260 own a van | 125 own a truck and a van | 20 own all three |
- What is the probability that a randomly chosen person owns exactly two of these types of cars?
 - What is the probability that a randomly chosen person owns a sedan and a truck but not a van?
 - What is the probability that a randomly chosen person does not own any of these types of cars?
 - What is the probability that a randomly chosen person owns at least one of these types of cars?
17. 9 people go out for a job, 3 women and 6 men. 3 people will be chosen for this job.
- What is the probability that exactly 2 women will be chosen?
 - What is the probability at least 2 women are chosen?
 - If we know exactly 2 women are chosen, what is the probability a man will be chosen for the third position?

- (d) If Sue and Dan are two people up for the job, what is the probability that Dan will get it if we know that Sue will get the job?
18. Five eggs are left in a carton. Two of them are rotten. You start cracking the eggs one at a time until you have cracked all rotten eggs or all good eggs.
- Draw the tree diagram to represent this process, including probabilities along the branches and at the final nodes.
 - What is the probability that you will have to crack 3 eggs?
 - What is the probability that you will have to crack 3 eggs given that you cracked a rotten egg first?
 - What is the expected number of rotten eggs you will crack?
19. 5 boys and 3 girls are going to sit in a row of 8 adjacent seats.
- In how many different ways can they seat themselves?
 - In how many different ways can they seat themselves if the 3 girls all sit together?
 - In how many different ways can they seat themselves if Ann and Bob sit together?
20. Mike plays checkers with his sister Mary. And he wins 80% of the time. If they play 7 games of checkers,
- What is the probability that Mike will win exactly 5 of the 7 games?
 - What is the probability that Mike will win at least 5 games?
 - What is the expected number of games that Mike will win?
21. A student takes a test 5 times. The probability of her failing the test each time is 0.24. What is the probability that she will pass the test at most 2 times?
22. A town has two doctors, Smith and King. Each year 20% of Smith's patients switch to Dr. King, and 40% of King's patients switch to Dr. Smith.
- Find the transition matrix T .
 - If 60% of the people in the town presently use Dr. Smith, what percentage of the residents will be using Dr. Smith after 2 years have passed?
 - In the long run what fraction of the patients will Smith have?
23. Patients with a certain disease are tracked each year and classified as sick, in remission, cured, or dead. Each year 30% of those who are sick remain sick, 40% go into remission, and 30% die. Each year 20% of those in remission are cured, 10% become sick again, and the remaining 70% stay in remission.
- Set this up as a 4-state Markov Chain with "cured" and "dead" being absorbing states. Give the transition matrix with clearly labeled rows and columns.
 - 50% the patients in a certain clinic are currently sick and 50% are in remission. Find the fraction of these patients that will be in each of the 4 categories after an additional year has passed. (hint: use the idea of an initial probability distribution.)
 - Suppose you want to know the probability that a person who is currently sick will eventually be cured. To calculate this probability requires finding the inverse of a matrix. Write down the matrix whose inverse would have to be found to solve this problem. You do not have to find the inverse or solve the problem.