p96 #4

Maximize $P = 2x - y + 4z$

subject to $3x + 2y + 4z \le 12$

$x + y \le 4$

$y + 2z \le 5$

$x \ge 0 \quad y \ge 0 \quad z \ge 0$

introduce slack variables

$3x + 2y + 4z \le 12 \implies 3x + 2y + 4z + u = 12$

$x + y \le 4 \implies x + y + v = 4$

$y + 2z \le 5 \implies y + 2z + w = 5$

objective function $P = 2x - y + 4z \implies -2x + y - 4z + P = 0$

enter in simplex tableau

$$\begin{array}{l} \\ \text{constraints}\left\{\begin{array}{l} \\ \\ \\ \end{array}\right. \\ \text{objective} \end{array} \begin{array}{c} \begin{array}{ccccccc|c} x & y & z & u & v & w & P & \text{const} \end{array} \\ \left(\begin{array}{ccccccc|c} 3 & 2 & 4 & 1 & 0 & 0 & 0 & 12 \\ 1 & 1 & 0 & 0 & 1 & 0 & 0 & 4 \\ 0 & 1 & 2 & 0 & 0 & 1 & 0 & 5 \\ -2 & 1 & -4 & 0 & 0 & 0 & 1 & 0 \end{array}\right) \end{array}$$

negative in last row of biggest magnitude is -4

since $|-4| > |-2|$

pivot is in z-column, need row

$$\begin{array}{c} \begin{array}{ccccccc|c} x & y & z & u & v & w & P & \end{array} \\ \left(\begin{array}{ccccccc|c} 3 & 2 & 4 & 1 & 0 & 0 & 0 & 12 \\ 1 & 1 & 0 & 0 & 1 & 0 & 0 & 4 \\ 0 & 1 & 2 & 0 & 0 & 1 & 0 & 5 \\ -2 & 1 & -4 & 0 & 0 & 0 & 1 & 0 \end{array}\right) \end{array}$$

ratios const col / pivot col

$\frac{12}{4} = 3$

doesn't count division by 0

$5/2 = 2\frac{1}{2}$

pivot never in last row

$2\frac{1}{2}$ smallest

$\therefore$ 3rd row

$$\begin{array}{c} \begin{array}{ccccccc|c} x & y & z & u & v & w & P & \end{array} \\ \left(\begin{array}{ccccccc|c} 3 & 2 & 4 & 1 & 0 & 0 & 0 & 12 \\ 1 & 1 & 0 & 0 & 1 & 0 & 0 & 4 \\ 0 & 1 & 2 & 0 & 0 & 1 & 0 & 5 \\ -2 & 1 & -4 & 0 & 0 & 0 & 1 & 0 \end{array}\right) \end{array}$$

2 in 3rd row 3rd col is pivot

$$\begin{array}{ccccccc|c} x & y & z & u & v & w & p & \text{const} \end{array}$$

$$\xrightarrow{\frac{1}{2}R_3} \left(\begin{array}{ccccccc|c} 3 & 2 & 4 & 1 & 0 & 0 & 0 & 12 \\ 1 & 1 & 0 & 0 & 1 & 0 & 0 & 4 \\ 0 & \frac{1}{2} & 1 & 0 & 0 & \frac{1}{2} & 0 & \frac{5}{2} \\ -2 & 1 & -4 & 0 & 0 & 0 & 1 & 0 \end{array}\right)$$

make rest of z column into zeros

$$\begin{array}{ccccccc|c} x & y & z & u & v & w & p & \text{const} \end{array}$$

$$\xrightarrow{R_1 - 4R_3} \left(\begin{array}{ccccccc|c} 3 & 0 & 0 & 1 & 0 & -2 & 0 & 2 \\ 1 & 1 & 0 & 0 & 1 & 0 & 0 & 4 \\ 0 & \frac{1}{2} & 1 & 0 & 0 & \frac{1}{2} & 0 & 5/2 \\ -2 & 1 & -4 & 0 & 0 & 0 & 1 & 0 \end{array}\right)$$

$$\begin{array}{ccccccc|c} x & y & z & u & v & w & p & \text{const} \end{array}$$

$$\xrightarrow{R_4 + 4R_3} \left(\begin{array}{ccccccc|c} 3 & 0 & 0 & 1 & 0 & -2 & 0 & 2 \\ 1 & 1 & 0 & 0 & 1 & 0 & 0 & 4 \\ 0 & \frac{1}{2} & 1 & 0 & 0 & \frac{1}{2} & 0 & 5/2 \\ -2 & 3 & 0 & 0 & 0 & 2 & 1 & 10 \end{array}\right)$$

basic solution $x = y = 0$ not basic $w = 0$ not basic

$z = \frac{5}{2}$ $u = 2$ $v = 4$ $P = 10$

But negatives in bottom row $\therefore$ not finished

Largest negative in bottom row is -2

$$\begin{array}{ccccccc|c} x & y & z & u & v & w & p & \text{const} \end{array}$$

$$\left(\begin{array}{ccccccc|c} 3 & 0 & 0 & 1 & 0 & -2 & 0 & 2 \\ 1 & 1 & 0 & 0 & 1 & 0 & 0 & 4 \\ 0 & \frac{1}{2} & 1 & 0 & 0 & \frac{1}{2} & 0 & \frac{5}{2} \\ -2 & 3 & 0 & 0 & 0 & 2 & 1 & 10 \end{array}\right)$$

ratio $\frac{\text{const col}}{\text{pivot col.}}$

$\frac{2}{3}$

$\frac{4}{1}$

division by zero, don't consider

$\frac{2}{3} < 4$ so 1^{st} row = pivot row

$$\begin{array}{ccccccc|c} x & y & z & u & v & w & p & \text{const} \end{array}$$

$$\left(\begin{array}{ccccccc|c} 3 & 0 & 0 & 1 & 0 & -2 & 0 & 2 \\ 1 & 1 & 0 & 0 & 1 & 0 & 0 & 4 \\ 0 & \frac{1}{2} & 1 & 0 & 0 & \frac{1}{2} & 0 & \frac{5}{2} \\ -2 & 3 & 0 & 0 & 0 & 2 & 1 & 10 \end{array}\right)$$

3 in row 1 col 1 is pivot

$$\xrightarrow{\frac{1}{3}R_1}\quad \begin{array}{c} \begin{array}{cccccccc} x & y & z & u & v & w & p & \text{const} \end{array} \\ \left(\begin{array}{ccccccc|c} 1 & 0 & 0 & \frac{1}{3} & 0 & -\frac{2}{3} & 0 & \frac{2}{3} \\ 1 & 1 & 0 & 0 & 1 & 0 & 0 & 4 \\ 0 & \frac{1}{2} & 1 & 0 & 0 & \frac{1}{2} & 0 & 5/2 \\ -2 & 3 & 0 & 0 & 0 & 2 & 1 & 10 \end{array}\right) \end{array}$$

$$\xrightarrow{R_2-R_1}\quad \begin{array}{c} \begin{array}{cccccccc} x & y & z & u & v & w & p & \text{const} \end{array} \\ \left(\begin{array}{ccccccc|c} 1 & 0 & 0 & \frac{1}{3} & 0 & -\frac{2}{3} & 0 & \frac{2}{3} \\ 0 & 1 & 0 & -\frac{1}{3} & 1 & \frac{2}{3} & 0 & \frac{10}{3} \\ 0 & \frac{1}{2} & 1 & 0 & 0 & \frac{1}{2} & 0 & \frac{5}{2} \\ -2 & 3 & 0 & 0 & 0 & 2 & 1 & 10 \end{array}\right) \end{array}$$

$$\xrightarrow{R_4+2R_1}\quad \begin{array}{c} \begin{array}{cccccccc} x & y & z & u & v & w & p & \text{const} \end{array} \\ \left(\begin{array}{ccccccc|c} 1 & 0 & 0 & \frac{1}{3} & 0 & -\frac{2}{3} & 0 & 2/3 \\ 0 & 1 & 0 & -\frac{1}{3} & 1 & 2/3 & 0 & 10/3 \\ 0 & \frac{1}{2} & 1 & 0 & 0 & \frac{1}{2} & 0 & 5/2 \\ 0 & 3 & 0 & 2/3 & 0 & 2/3 & 1 & \frac{34}{3} \end{array}\right) \end{array}$$

No negatives in bottom row $\therefore$ DONE!

basic solution = optimal solution

y, u, w NOT basic so $y = u = w = 0$

$$\begin{array}{c} \begin{array}{cccccccc} x & y=0 & z & u=0 & v & w=0 & p & \text{const} \end{array} \\ \left(\begin{array}{ccccccc|c} 1 & 0 & 0 & \frac{1}{3} & 0 & -\frac{2}{3} & 0 & 2/3 \\ 0 & 1 & 0 & -\frac{1}{3} & 1 & \frac{2}{3} & 0 & 10/3 \\ 0 & \frac{1}{2} & 1 & 0 & 0 & \frac{1}{2} & 0 & 5/2 \\ 0 & 3 & 0 & 2/3 & 0 & \frac{2}{3} & 1 & \frac{34}{3} \end{array}\right) \end{array}$$

ignore   ignore   ignore

$x = \frac{2}{3}$ $z = 5/2$ $v = \frac{10}{3}$ $p = \frac{34}{3}$ maximum value

$3\overline{)34}$ = $11\frac{1}{3}$ $11\frac{1}{3} = 11.\overline{3}$