

ECG790C - Problem Set 6 - Spring 2009

1. Consider the following functional equation

$$V(S) = f(S) + \beta \int V(z)p(z; S)dz$$

where f and p are known functions. Specifically, let $f(S) = S$ and $p(z; S)$ be Beta density function with $E[z|S] = (S + 0.5)/2$ and $Var[z|S] = 1/12 + (S - 0.5)^4$. Recall that the Beta density is

$$B(z; a, b) = \frac{z^{a-1}(1-z)^{b-1}}{B(a, b)}$$

where $B(a, b) = \Gamma(a)\Gamma(b)/\Gamma(a+b)$ is the Beta function and that $E[z] = a/(a+b)$ and $Var(z) = ab/((a+b)^2(a+b+1))$.

Write a Matlab script that computes and plots a polynomial approximation to V , with $\beta = 0.9$. Use some numerical quadrature method to approximate the integral. Better yet, approximate V with polynomials, splines and piecewise linear functions and the integral with trapezoid, Simpson's, Gauss-Legendre and the modified Gauss-Jacobi method implemented in `qnwbeta` and then discuss the differences.

Note that there is a closed form solution to this problem but this should only be used to check your answer (if you want to use it at all). The solution is affine in S ; the coefficients of the affine function can be determined by substitution into the functional equation and solving.

To solve this problem numerically you need first to determine the values of the Beta distribution parameters as functions of S . Then you need to write code that will compute the appropriate quadrature nodes and weights and the basis functions for the approximation of V . Finally you need to solve for the coefficients of the approximation of V .